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Competition and Externalities in Green Technology Adoption

Maxime C. Cohen, Georgia Perakis,
and Charles Thraves

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Maxime C. Cohen

Operations Research Center, MIT, Cambridge, MA 02139, maxcohen@mit.edu

Georgia Perakis

Sloan School of Management, MIT, Cambridge, MA 02139, georgiap@mit.edu

Charles Thraves

Operations Research Center, MIT, Cambridge, MA 02139, cthraves@mit.edu

In this paper, we study the effects of competition among multiple suppliers who sell green technology products, such as electric vehicles. The government offers consumer subsidies to encourage the product adoption. We consider a setting where suppliers adjust production and price depending on the level of subsidies offered by the government to the consumers. Our analysis expands the understanding of symmetric and asymmetric competition, incorporating the external influence from the government who is now an additional player in the system. We quantify how competition impacts the consumers, the suppliers as well as the government relative to the monopolistic setting where all the products are jointly produced from a single firm. In other words, we quantify who is benefitting from the competition and under what conditions. Our model incorporates demand uncertainty as well as positive externalities. We first compare different government objectives and determine that the magnitude of the externalities plays a key role in selecting the right objective. We then show that the effects of competition may differ depending on the demand uncertainty, the supplier asymmetry and the magnitude of the externalities. When externalities are relatively small, we show that competition hurts the suppliers and benefits the government. However, it does not always benefit all the consumers, as it is usually the case in classical competition settings. We also show that in a market with large externalities, consumers, unlike the government, are always better-off in a competitive environment. Finally, we test our model and validate our insights using actual data from the electric vehicle industry, which is becoming increasingly competitive.

Key words: Competition, Externalities, Government Subsidies, Green Technology Adoption, Newsvendor

1. Introduction

1.1. Motivation

Global warming has continuously increased during the last decades, bringing many undesired consequences to Earth and human life. Predictions suggest the consequences will continue to worsen over the years to come. One of the solutions to mitigate this problem is the adoption of green technologies. As a result, this prospect has captured the attention of public and private sectors. However, green products (such as electric vehicles and solar panels) remain usually unaffordable

and many people continue using conventional ones. In order to overcome this issue and encourage green technology adoption, several governments started to offer subsidies (or tax rebates) to consumers in order to enhance the adoption of these technologies.

In the height of the economic recession, the US government passed the American Recovery and Reinvestment Act of 2009 which granted a tax credit for consumers who purchased an EV. Besides boosting the US economy, this particular tax incentive was aimed at fostering further research and economies of scale in the nascent electric vehicle industry. Indeed, sales of EVs in the US have effectively been increasing throughout the past five years¹.

Growing demand in these markets has attracted the interest of different car manufacturers. Indeed, in December 2010, the all-electric car, Nissan Leaf, and the plug-in hybrid General Motors' Chevy Volt were both introduced in the US market. After a slow first year, sales started to pick up and most major car companies are now in the process of launching their own versions of electric vehicles². Even if we restrict to the highway capable vehicles (i.e., road cars with a top speed above 65 mph), one can count above 20 models of EVs available in the market (December 2014).

As we previously mentioned, General Motors and Nissan have recently introduced affordable electric vehicles in the US market. GM's Chevy Volt was awarded the most fuel-efficient compact car with a gasoline engine sold in the US, as rated by the United States Environmental Protection Agency (EPA (2012)). However, the price tag of the Chevy Volt is still considered high for its category. The cumulative sales of the Chevy Volt in the US since it was launched in December 2010 until December 2014 amount to 73,357. It is likely that the \$7,500 government subsidy offered to each buyer through a federal tax credit played a significant role in the sales volume. The manufacturer's suggested retail price (MSRP) of GM's Chevy Volt in April 2015 was \$34,995 but the consumer was eligible for \$7,500 tax rebates so that the effective price reduced to \$27,495. The size of consumer subsidy has remained constant since launch in December 2010 until the end of 2014. In addition, most other EVs are eligible for the same \$7,500 tax rebate, so that consumers can choose between the different vehicles while still receiving the same subsidy from the government.

In this paper, we address the following questions: How does the recent competition in the EV industry affect consumer subsidies and green technology adoption? How should governments take into account competition effects while designing consumer subsidies for green technology adoption? Finally, how does competition affect the suppliers' prices (MSRP), production quantities and consumers? Note that competition can either be symmetric (with perfectly substitutable products) or asymmetric (with different products, such as family versus luxury cars). In this paper, we consider both cases and study how a monopolistic setting differs from a competitive environment. By

¹ <http://electricdrive.org/index.php?ht=d/sp/i/20952/pid/20952>

² <http://energy.gov/articles/visualizing-electric-vehicle-sales>

understanding the impact of competition in the green technology market, the government can then design more efficient subsidy programs.

Externalities are considered as the monetary value of the reduction in CO₂ emissions of an EV versus a regular car. We consider two types of markets: small and large externalities. We show that the analysis as well as some of the main insights might be different depending of the level of externalities. Consequently, the impact of the competition is highly affected by the externality factors. By externalities, we refer to environmental benefits from green technology adoption. For example, each electric vehicle sold provides a reduction in CO₂ emissions which the government may want to consider while designing consumer subsidies.

In this paper, we introduce a model to study the effects of competition in green technology markets with substitutable products and uncertain demand, such as EVs. We show that externalities, demand uncertainty and suppliers' asymmetry all play a key role in answering this question and we derive various insights regarding the impact of competition for green technology adoption.

1.2. Contributions

Given the recent growth of green technologies, supported by governmental subsidy programs, this paper explores a timely problem in supply chain management. Understanding how the recent increasingly competition in this industry affects subsidy costs, as well as the economic surplus of suppliers and consumers, is an important part of designing sensible subsidy programs. The main contributions of this paper are:

- *Should the government maximize social welfare or minimize expenditures?*

We compare the outcomes when the government optimizes social welfare versus expenditures. We derive analytical tight bounds on the social welfare loss when the government minimizes expenditures, and investigate what is the most appropriate objective for the government. In particular, we show that maximizing social welfare can be very costly to the government. When externalities are small, we show that the government can minimize expenditures and still attain a very good level of welfare while reducing significantly the expenditures. Nevertheless, when externalities are large the social welfare loss may be rather significant.

- *The interplay of demand uncertainty and suppliers asymmetry determines the repartition of the competition benefit.*

We show that the benefit from the presence of competition is shared among the different players. For example, in a market with small externalities, the suppliers are always worse off in a competitive setting and the benefit is shared between the government and the consumers. Moreover, the exact sharing of this benefit depends on the interplay of demand uncertainty and supplier asymmetry. We show that when demand is deterministic and suppliers are identical, the entire benefit is absorbed

by the government. We also determine that when suppliers are asymmetric, competition does not benefit all the consumers. Finally, demand uncertainty favors consumers in terms of competition benefit.

- *Competition always benefits the government and hurts suppliers, when externalities are small.*

As expected, we show that when externalities are small, competition always hurts the suppliers, as is usually the case. We also observe that competition always favors the government by allowing a reduction in expenditures. This result indicates that the government should encourage competition and incentivize new entrants to the electric vehicle industry.

- *Competition always benefits consumers, when externalities are large.*

By considering a market with large externalities, we show that the effects of competition may differ. In particular, it is not clear anymore that the government is better off at the expense of the suppliers. In this case, competition always benefits consumers who enjoy (for a symmetric setting) higher available quantities and a lower effective price.

1.3. Literature review

Consumer subsidies for green technologies have been an active area of research for the last two decades (for recent works see, e.g., Cohen et al. (2015a), Ovchinnikov and Raz (2013), Chemama et al. (2015) and Zhang (2014)). Zhang (2014) analyzes the impact of subsidies on risk averse EV manufacturers in a newsvendor setting by looking at the trade-off between subsidy levels, risk aversion, demand uncertainty and performance. Similar to this paper, other works incorporate the government as a player who decides a policy typically in the first stage, followed by the response from a firm. An example includes Cohen et al. (2015a), where the authors consider a monopolistic firm and investigate the impact of demand uncertainty on subsidy policies for non-linear stochastic demands. The authors consider a cost minimization objective where the government offers consumer subsidies in order to achieve an adoption target level set by the government. Taking a slightly different perspective, several authors look into a social welfare maximization approach from the government. Ovchinnikov and Raz (2013) compare, for the case of linear demand, different government intervention mechanisms and investigate under what conditions the system is coordinated in terms of welfare, prices and supply quantities. Our paper considers a general framework where multiple competing manufacturers are present in the market that can include externalities. Indeed, motivated by recent developments in the EV industry, several car manufacturers started offering their own electric vehicle. Therefore, it seems appealing to study how competition affects the outcomes of the various players involved. In addition, we consider and compare both government objectives: maximizing the social welfare versus minimizing the government cost.

Another line of research related to this paper aims to quantify the positive externalities generated by green technology products, such as solar panels and electric vehicles. In particular, several works have studied the monetary impact of the different pollutants in a dollar base, (see, e.g., Matthews and Lave (2000)). Other works have studied the efficiency of green technologies by comparing the impact of these products versus the impact of regular ones. For instance, how much more efficient is an EV relative to a similar regular fuel vehicle? (see, e.g., Lave and MacLean (2002)). Additional works consider not only the externalities produced by the reduced emissions from EVs, but also the different sources of energy used to charge the vehicles. For example, Arar (2010) studies the mix of energy sources in the US. Holdway et al. (2010) compare sources of energy in different countries, concluding that EVs actually reduce CO₂ emissions regardless of the sources of energy consumed. However, the more renewable the sources of energy are, the greater the benefit of EVs. Carlsson and Johansson-Stenman (2003) examine the social benefits of electric vehicle adoption in Sweden and report a pessimistic outlook for this technology in the context of net social welfare. Avci et al. (2013) show that the adoption of electric vehicles has societal and environmental benefits, as long as the electricity grid is sufficiently clean. This paper assumes non-strategic industry players. In our paper though, we incorporate the strategic response of the industry into the policy making decision.

Without considering demand uncertainty, there is a significant amount of empirical work in the economics literature on the effectiveness of subsidy policies for hybrid and electric vehicles. For example, Diamond (2009) shows that there is a strong relationship between gasoline prices and hybrid adoption. Chandra et al. (2010) show that hybrid car rebates in Canada created a crowding out of other fuel efficient vehicles in the market. Gallagher and Muehlegger (2011) argue that sales tax waivers are more effective than income tax credits for hybrid cars. The increase in hybrid car sales from 2000-2006 is mostly explained by social preferences and increasing gasoline prices. Aghion et al. (2012) show that the auto industry innovates more in clean technologies when fuel prices are high.

Our methodology is related to the newsvendor problem, an extensively studied problem in the literature (see, e.g., Porteus (1990) and the references therein). Numerous extensions have been subsequently proposed. For example, Petruzzi and Dada (1999) and Yao et al. (2006) consider a price-setting newsvendor for a single supplier with additive and multiplicative noises. Choi (2012) also presents an overview of different facets of the newsvendor problem. Similar to our paper, it studies the price-setting newsvendor for multiple substitute products with a linear demand (Chapter 1.4.1 Choi (2012)). In our setting, we model the problem as a Stackelberg game, where in the second stage, the suppliers are price-setting competing newsvendors responding to the subsidy announced by the government in the first stage.

The issue of how demand uncertainty creates a mismatch in supply and demand has been well studied in the literature. For example, Sallee (2011) argues that consumers captured most of the incentives for the Toyota Prius, while the firm cannot capture any of this surplus despite a binding production constraint. The author also shows that there was a shortage of vehicles manufactured to meet demand when the Prius was launched. This reinforces our motivation for considering a newsvendor model in this context.

An additional stream of works on a more qualitative base of EV adoption have shown that most people primarily care about costs, range and performance of vehicles (e.g., Graham-Rowe et al. (2012)). In Lieven et al. (2011), the authors propose a way to forecast future sales of EVs in Germany and conclude that the two main barriers remain the price and the range. Indeed, most people do not consider the positive environmental effects of green technologies as their primary driver of the decision to purchase a green product: the features and price are more relevant, (see Caperello and Kurani (2012)). Consequently, consumer subsidies might play a key role in EV adoption. Therefore, governments should design these subsidies with care. Studying the effects of competition in a model that includes network externalities and demand uncertainty in this context is the main motivation of this paper.

Structure of the paper

In Section 2, we describe the model and assumptions we impose. In Section 3, we discuss the difference between two common government objectives: maximizing welfare versus minimizing expenditures. In Section 4, we study the effects of competition in markets with small and large externalities. In Section 5, we present some computational experiments using actual data from the electric vehicle industry in order to test and validate our model and insights. Finally, we present our conclusions in Section 6. Most of the proofs of the different propositions and theorems are relegated to the Appendix.

2. Model and Assumptions

Consider a green technology market for which the government designs a consumer subsidy in order to encourage technology adoption. In particular, we assume that the government aims to achieve a target adoption level, denoted by Γ . The government offers a uniform subsidy (or rebate) r directly to the end consumers. In the second stage, n suppliers follow by deciding quantities q_i and prices p_i ; $\forall i = 1, 2, \dots, n$ using a price-setting newsvendor model. In other words, the market is composed of n substitutable green technology products (e.g., electric vehicles), where each product $i \in \{1, \dots, n\}$ has a marginal production cost c_i . Consumer demand is modeled as an affine function with additive uncertainty as follows:

$$d = \bar{d} - B(p - re) + \epsilon, \tag{1}$$

where $e \in \mathbb{R}^n$ is a vector of ones and $\epsilon \in \mathbb{R}^n$ is a random vector with components ϵ_i . We denote by $F_i(\cdot)$ and $f_i(\cdot)$ the cumulative and density distribution functions respectively, for product i .

ASSUMPTION 1. *We impose the following conditions on demand.*

- ϵ_i are independent, with bounded support $[-A_i, A_i]$ and zero mean.
- The noise distributions have increasing failure rate (IFR), (i.e., $f_i(x)/(1 - F_i(x))$ is a non-decreasing function).
- The noises satisfy $\bar{d} - Bc \geq A$ to ensure that demand is non-negative for each noise realization.
- The price elasticity matrix $B \in \mathbb{R}^{n \times n}$ is symmetric, strictly diagonal dominant and an M-Matrix³.

The symmetry requirement of the matrix B follows from the Slutsky condition (see, e.g., Farahat and Perakis (2010) and Krishnan (2010)), which essentially states that the demand function is derived from an underlying concave utility function of a representative consumer. The M-Matrix property is driven by the sign of the price elasticities in order to model a market with substitutable goods. More precisely, demand for a particular product is a decreasing function of its own price and non-decreasing with respect to prices of competitive products. The diagonal dominance condition captures the fact that the net effect of a variation in the self price is more significant than the same variation in all the competitive prices. Finally, we impose the natural assumption that the target adoption level cannot be attained with zero rebate (otherwise, the problem is irrelevant).

Regarding the government objective, we consider two of the most common objective functions in the literature: minimizing government cost and maximizing social welfare. The Social Welfare (SW) is defined as the total system surplus, which includes the Firms' Profits (FP), plus the Consumer Surplus (CS), minus Government Cost (GC) plus positive Externalities (EX):

$$SW = FP + CS - GC + EX. \quad (2)$$

Our goal is to study the effects of competition. In particular, we compare a monopolistic setting to a competitive environment, both with n green technology products. In both cases, the total profits of the suppliers can be expressed as:

$$\Pi = p' \min\{d, q\} - c'q,$$

where, p and q are the vectors of price and production quantities chosen by the suppliers, c is the vector of manufacturing costs and d is the uncertain demand vector.

Finally, we characterize the consumers by measuring the consumer surplus, that is a common metric to capture consumer satisfaction. More precisely, the consumer surplus is defined as the

³ $B \in \mathbb{R}^n$ is an M-Matrix if and only if $B_{ii} \geq 0 \forall i \in \{1, \dots, n\}$ and $B_{ij} \leq 0 \forall i, j \in \{1, \dots, n\} i \neq j$.

difference between the maximum price a consumer is willing to pay and the actual market price. We note that in our case, the market price is equal to the effective price paid by the consumers, i.e., $z = p - r$. When demand is deterministic, we denote by $D^{-1}(q)$ the effective price that will generate demand exactly equal to q . The consumer surplus is given by: $CS = \int_z^{z_{max}} D(z)dz$, where z_{max} corresponds to the value of the effective price that yields zero demand.

When demand is uncertain however, defining the consumer surplus is somewhat more subtle due to the possibility of a stock-out. Several papers on peak load pricing and capacity investments by a power utility under stochastic demand address partially this modeling issue (see Carlton (1986), Crew et al. (1995) and Brown and Johnson (1969)). Nevertheless, the models developed in this literature are not applicable to the price-setting newsvendor. More specifically, in Brown and Johnson (1969) the authors assume that the utility power facility has access to the willingness to pay of the customers so that it can decline the ones with the lowest valuations. This assumption is not justifiable in our setting where a “first-come-first-serve” logic with random arrivals is more suitable. In Ovchinnikov and Raz (2013), the authors study a price-setting newsvendor model for public goods and consider the consumer surplus for linear additive stochastic demand. In Cohen et al. (2015b), the authors extend the treatment of the consumer surplus for a general framework that includes non-linear demands for multiple products and consider various rationing capacity rules.

For general stochastic demand functions, the consumer surplus $CS(\epsilon)$ is defined for each realization of demand uncertainty ϵ . If there was no supply constraint, considering the effective price and the realized demand, the total amount of potential consumer surplus is defined as: $\int_{z_{sto}}^{z_{max}(\epsilon)} D(z, \epsilon)dz$. Since customers are assumed to arrive in a first-come-first-serve manner, irrespective of their willingness to pay, some proportion of these customers will not be served due to stock-outs. The proportion of served customers is given by the ratio of actual sales over potential demand: $\frac{\min(D(z_{sto}, \epsilon), q_{sto})}{D(z_{sto}, \epsilon)}$. Therefore, the consumer surplus can be defined as the total available surplus times the proportion of that surplus that is actually served, i.e.,

$$CS(\epsilon) = \int_{z_{sto}}^{z_{max}(\epsilon)} D(z, \epsilon)dz \cdot \frac{\min(D(z_{sto}, \epsilon), q_{sto})}{D(z_{sto}, \epsilon)}. \quad (3)$$

We note that in this case, Consumer Surplus is a random variable that depends on the demand uncertainty through the noise ϵ . Note that we are interested in computing the expected consumer surplus $\mathbb{E}_\epsilon[CS(\epsilon)]$. For stochastic demand, (3) has a similar interpretation as its deterministic counterpart. Nevertheless, we also incorporate the possibility that a consumer who wants to buy the product does not find it available. For the linear demand function in (1), the expected consumer surplus is given by:

$$CS = \frac{1}{2} \min\{d, q\}' B^{-1} d. \quad (4)$$

Finally, the last factor included in the social welfare comes from the externalities. Given that green technologies have a positive environmental impact, we consider that each unit sold of product i induces a positive externality factor k_i on the society (for example, due to reduction in emissions). Therefore, the total expected externalities are given by:

$$EX = k' \min\{d, q\}, \quad (5)$$

where k is the vector with components k_i ; $\forall i = 1, 2, \dots, n$.

The alternative objective of the government is to simply minimize the total expenditures, given by:

$$GC = re' \min\{d, q\}. \quad (6)$$

We assume that the government is setting a rebate level r that is identical across all the different products. This assumption is justified for example in the EV market, where consumers are eligible for a tax rebate that amounts to \$7,500 for the vast majority of the cars. In this paper, we study and compare both government objectives (2) and (6). Finally, we impose the following assumption on demand in order to ensure the concavity of the supplier's problem.

ASSUMPTION 2. *We impose the following condition on demand: $\frac{1}{f_i(-A_i)c_i} < e_i B e$; $\forall i \in \{1, \dots, n\}$, where $c_i > 0$ and $A_i > 0$.*

Assumption 2 states that the price sensitivity matrix B has to be strictly diagonal dominant so that the self elasticities B_{ii} outweigh the sum of the cross elasticities by a factor of $1/(f_i(-A_i)c_i)$. This factor accounts for the demand uncertainty and ensures that the effect of the noise does not overcome the diagonal dominance condition for substitutable goods. More precisely, the magnitude of the noise should not be too large, such that the density evaluated at the lowest noise realization is bounded away from zero. In addition, Assumption 2 is a sufficient condition to guarantee the concavity of the problem faced by the suppliers (in both the monopolistic and competitive settings). As a result, this ensures the existence and uniqueness of an equilibrium. Finally, we note that this assumption is easily satisfied for practical settings, as we will show in Section 5.

2.1. Monopolistic suppliers

In this section, we consider a single firm that jointly manages all the different products. More precisely, the monopolist decides prices and production quantities for the n substitutable green technology products (e.g., n different versions of electric vehicles). In the alternative setting considered in the next section, we assume that each product is managed by a different supplier, leading to a competitive environment. Our goal is to compare the outcomes in both settings in order to study

the effects of competition on the various players involved (government, suppliers and consumers). In the second stage, the firm sets its production quantities and prices for a given rebate level r set by the government, so as to maximize total expected profit. We define the following function:

$$\Psi_i(p_i) = \mathbb{E} \left[\min \left(\epsilon_i, F_i^{-1} \left(1 - \frac{c_i}{p_i} \right) \right) \right], \quad (7)$$

which represents the negative of the expected shortages evaluated at the optimal newsvendor quantities. Note that since ϵ has zero mean, $\Psi_i(p_i) \leq 0$. Therefore the profit maximization problem can be written as:

$$\max_{p,q} p' \mathbb{E}[\min\{d, q\}] - c'q. \quad (8)$$

In order to keep the notation compact, we denote $F^{-1}(1 - c/p)$ an n dimensional function with components $F_i^{-1}(1 - c_i/p_i) \quad \forall i \in \{1, \dots, n\}$. In equation (8), we used the fact that the optimal production $q^*(p)$ as a function of the price is given by:

$$q^*(p) = \bar{d} - B(p - er) + F^{-1}(1 - c/p), \quad (9)$$

from the first order condition. We next characterize the optimal price response of the monopolist.

PROPOSITION 1. *Under Assumption 2, for a given rebate level r , the unique solution of problem (8) is given by $p \in \mathbb{R}^n$ that solves the following fixed point system of equations:*

$$p^N(r) = \frac{1}{2} B^{-1} \left(\bar{d} + Ber + Bc + \Psi(p^N(r)) \right). \quad (10)$$

Proof. See Appendix A. \square

Note that in order to find the optimal price for a given r , one needs to solve a non-linear system of fixed point equations. In Appendix A, we show that there exists a unique solution to this system. Note that if Assumption 2 is not satisfied, the problem is not necessarily concave but is still numerically tractable (see Petruzzi and Dada (1999)). After computing the optimal price $p^N(r)$ from (10), one can derive the optimal quantity vector using (9). As expected, all the components of the optimal price vector are increasing with respect to r , which is formally shown in Appendix B.

In the first stage, the government decides the uniform rebate r to be offered to consumers. The government aims to achieve a target adoption level Γ in expectation. We consider and study two different government objectives: minimizing expenditures and maximizing social welfare, defined in (6) and (2) respectively.

- Minimizing Government Cost: In this case, the government faces the following optimization problem:

$$\begin{aligned} & \min_r \mathbb{E}[GC] \\ \text{s.t.} \quad & \mathbb{E} \left[e' \min\{q^N(r), d\} \right] \geq \Gamma. \end{aligned} \quad (11)$$

Given the firm's second stage response $p^N(r)$ and $q^N(r)$, the expected government cost is given by $\mathbb{E}[GC] = re' \left(\bar{d} - B(p^N(r) - er) + \Psi(p^N(r)) \right)$. We next show that the optimal solution of (11) can be computed by taking advantage of the monotonicity properties of the problem.

PROPOSITION 2. *Under Assumption 2, problem (11) has a unique optimal solution. In addition, this optimal solution is such that the adoption constraint is exactly met.*

Proof. See Appendix B. \square

Since the adoption constraint is monotonic with respect to r , Proposition 2 implies that the optimal rebate can be computed using a binary search on r such that $e' \left(\bar{d} - B(p^N(r) - er) + \Psi(p^N(r)) \right) = \Gamma$. We next discuss the second alternative government objective.

- Maximize Social Welfare: In this case, the government faces the following optimization problem:

$$\begin{aligned} & \max_r \mathbb{E}[SW] \\ & \mathbb{E} \left[e' \min\{q^N(r), d\} \right] \geq \Gamma \end{aligned} \tag{12}$$

As we previously discussed, the expected social welfare is given by:

$$\mathbb{E}[SW] = p' \mathbb{E} \left[\min\{d, q\} \right] - c'q - re' \mathbb{E} \left[\min\{d, q\} \right] + k' \mathbb{E} \left[\min\{d, q\} \right] + \frac{1}{2} \mathbb{E} \left[\min\{d, q\}' B^{-1} d \right].$$

We note that while solving problem (12), it is not clear anymore that the optimal solution is obtained by the tightness of the adoption constraint, as it was the case for problem (11). In particular, one can show that if the target level Γ is large enough, one can show that the adoption constraint is tight at optimality. Consequently, problems (11) and (12) yield the same optimal solution and thus are equivalent. However, if the target adoption is below a certain threshold value, the adoption constraint is not tight for problem (12) and the outcomes of both models will be different. One can characterize the threshold value depending on the parameters of the model. However, we will focus on the case where the target adoption is below the threshold in order to compare both government objectives in Section 3.

2.2. Competing suppliers

In this section, we consider a competitive environment where each firm $i \in \{1, \dots, n\}$ is in charge of a single product. In particular, each supplier decides the price and production for his product by maximizing his own profit without knowing about the other supplier's decisions. We model this scenario by n suppliers competing in a price-setting newsvendor with substitutable products, where each firm faces its own market (modeled by a price-demand stochastic function). As in the

previous case, in the second stage, each firm decides upon (p_i, z_i) so as to maximize its own profit. The optimization problem of each supplier can be formulated as:

$$\begin{aligned} & \max_{p_i, q_i} p_i \mathbb{E} \left[\min\{d_i, q_i\} \right] - c_i q_i \\ \Leftrightarrow & \max_{p_i} p_i e'_i \left(\bar{d} - B(p - er) + \Psi(p) \right) - c_i e'_i \left(\bar{d} - B(p - er) + F^{-1} \left(1 - \frac{c_i}{p} \right) \right). \end{aligned} \quad (13)$$

As before, we have used the fact that the optimal production as a function of the price $q^*(p)$ is given by (9). We denote by D the matrix with diagonal elements of B and zero elsewhere and $X = (B + D)^{-1}$.

PROPOSITION 3. *Under Assumption 2, for a given rebate level r , the unique solution of problem (13) is given by $p \in \mathbb{R}^n$ that solves the following fixed point system of equations:*

$$p^W(r) = X \left(\bar{d} + Ber + Dc + \Psi(p^W(r)) \right). \quad (14)$$

Proof. See Appendix C. \square

As in the previous case, after obtaining $p^W(r)$, the vector of quantities can be computed by equation (9). As before, one can see that the optimal prices are increasing in the rebate.

In the first stage, we consider the two different government objectives with the expected target adoption constraint (problem (11) and (12)), where the suppliers solve now (13) instead of (8). One can show similar results as in the monopolistic setting. In particular, under Assumption 2, the optimal solution of the government minimizing expenditures, is such that the target adoption is exactly met (see Appendix I). In addition, a similar result on a threshold value holds for the social welfare maximization.

3. Government Objectives: Comparisons

On one hand, in many economics contexts, researchers consider that governments aim to maximize social welfare. On the other hand, various operational models were proposed where the government seeks to minimize expenditures. In this section, we are comparing the two objectives for a competitive market with green technology products. Note that maximizing social welfare takes explicitly into account the entire system utility and therefore can be a desirable objective for the government. However, in many practical situations, policy makers are questioning the social welfare concept as it can be hard to measure and interpret. In addition, it is not clear that the government wants to incorporate supplier surplus as the firms are already optimizing it. Similarly, the consumers are already offered rebates, so the government already takes care of the consumers. Another potential issue with maximizing social welfare can be the high cost to achieve this objective. Minimizing government expenditures appears to be a more realistic and practical approach for the decision

makers, as it is easy to interpret and tries to save money for the government. However, one might wonder if it could result in a significant loss in terms of total welfare.

Recall that we have shown in the previous section, that when the target adoption level Γ is larger than a certain threshold, both problems yield the same outcome at optimality and hence are equivalent. Therefore, we focus on cases when the target adoption level is below the threshold and compare the outcomes from both problems. In order to compare the outcomes under the two different objectives, we study how the government cost and social welfare compare. In other words, if the government decides to minimize expenditures, how far is the social welfare from the optimal value? Similarly, if the government decides to maximize social welfare, how far is the government cost from the optimal value? We address these two questions for both the monopolistic setting and the competitive environment, denoted by superscripts N and W respectively.

We denote by GC_{SW} and GC_{GC} the resulting government cost when the government maximizes social welfare and minimizes cost respectively. Similarly, we denote by SW_{SW} and SW_{GC} the resulting social welfare when the government maximizes social welfare and when the government minimizes cost respectively. Our goal is to characterize the ratios $\frac{GC_{SW}}{GC_{GC}}$ and $\frac{SW_{GC}}{SW_{SW}}$ in order to compare both government objectives.

We also denote by p_0^N the vector of prices in the monopolistic setting, when the rebate is set to zero ($r = 0$) (and similarly p_0^W for the competitive environment). Let D_γ be the diagonal matrix with non negative entries $(\gamma_1, \dots, \gamma_n)$, where $\gamma_i = \frac{k_i}{p_{0,i}^N - c_i}$ for the monopolistic case, and $\gamma_i = \frac{k_i}{p_{0,i}^W - c_i}$ for the competition case. Recall that k_i represents the externality factor of product i , from (5). Finally, we define $\underline{\gamma} = \min_{i \in \{1, \dots, n\}} \{\gamma_i\}$ and $\bar{\gamma} = \max_{i \in \{1, \dots, n\}} \{\gamma_i\}$. In other words, γ_i represents the externality factor of product i normalized by the lowest profit margin. We first consider a deterministic demand and derive closed form expressions for both ratios. We then discuss the case where demand is uncertain. The results for the monopolistic setting are presented in the following Proposition.

PROPOSITION 4. 1. *Consider a monopolistic firm with deterministic demand. Then, we have:*

$$\frac{3 + 2\underline{\gamma}}{4 + 2\underline{\gamma} + 2\bar{\gamma} + \bar{\gamma}^2} \leq \frac{SW_{GC}^N}{SW_{SW}^N} \leq 1$$

$$M < \frac{GC_{SW}^N}{GC_{GC}^N} \quad \forall M > 0$$
(15)

Moreover, this bound is asymptotically tight.

2. *Consider a competitive setting with deterministic demand. Then, we have:*

$$\frac{3 + 2\bar{\gamma}}{(2 + \bar{\gamma})^2} \leq \frac{SW_{GC}^W}{SW_{SW}^W} \leq 1$$

$$M < \frac{GC_{SW}^W}{GC_{GC}^W} \quad \forall M > 0$$
(16)

Moreover, this bound is asymptotically tight, for $n = 2$.

Proof. See Appendix D. \square

Proposition 4 suggests that if the government focuses on maximizing social welfare, then the resulting expenditures may be arbitrarily large relative to the optimal government cost. Consequently, if the government has a given budget for its subsidy program, maximizing the social welfare may induce a solution that is not budget feasible, as the cost may be unbounded. This analysis potentially supports the fact that policy makers do not often seek to maximize social welfare. In addition, when the government minimizes expenditures, the resulting social welfare is not too far from the optimal value. For instance, when externalities are not present (i.e., $k = 0$), the resulting social welfare obtained is at most 25% from the optimal value. Note that this theoretical guarantee holds for all instances under this class of demand models. For many practical settings, the ratio is very close to one when externalities are not significant (as we will show computationally in Section 5). In conclusion, for the case where externalities are small, the government should minimize expenditures. It will result in significant savings relative to maximizing social welfare and still attain a near optimal welfare value.

The previous analysis was performed under the assumption that demand is deterministic. Indeed, when demand is stochastic (by incorporating an additive noise), one cannot characterize the ratios in closed form anymore. However, through extensive numerical testing, we observed that the results of Proposition 4 were preserved when demand is also stochastic. More specifically, we optimized over the parameters of the problem in order to find the minimal ratios (for both social welfare and government cost), using various noise distributions such as, uniform and truncated normal. The optimization always yields the worst case ratios for the case with a zero standard deviation, i.e., when demand is deterministic. Therefore, it seems that the results in Proposition 4 are also valid when demand is stochastic, since the deterministic case yields the worst case. Figure 1 presents the values of the ratios in the monopolistic setting for a particular instance with small externalities, as a function of the standard deviation. One can see that both ratios improve (get closer to one) as the standard deviation increases. In addition, for small standard deviations, minimizing government cost guarantees a near optimal social welfare (in this case, the social welfare ratio is between 0.8 and 0.98), while reducing the budget significantly.

However, as one can see from the results of Proposition 4, the social welfare ratio diverges relative to its optimal value, as externalities become more significant. In the limiting case, when the externality factor of one of the products approaches infinity, the lower bound on the social welfare ratio becomes arbitrarily close to zero. As a result, when externalities are large, the government should not minimize the expenditures anymore, as the loss in welfare may be very large. Unfortunately, maximizing social welfare is still not a desirable option as it can be very costly. For this reason, we introduce an alternative government objective, which can be seen as an intermediate

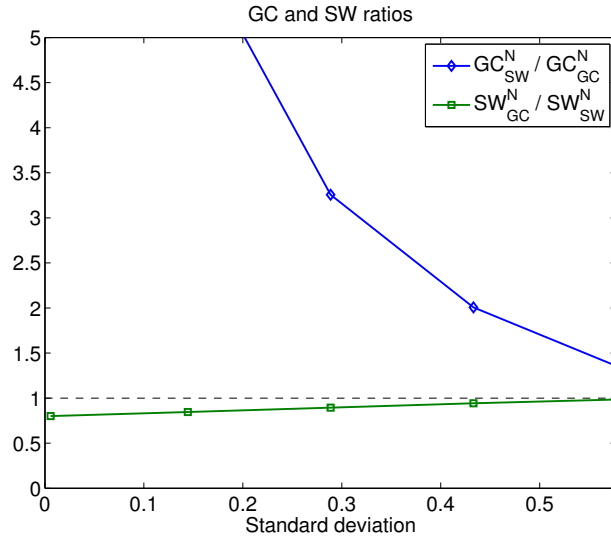


Figure 1 $\bar{d}_1 = 3.0$, $\bar{d}_2 = 2.8$, $B_{11} = 1.0$, $B_{12} = B_{21} = -0.05$, $B_{22} = 1.0$, $c_1 = 2.0$, $c_2 = 2.0$, $\epsilon_1, \epsilon_2 \sim U$, $\Gamma = 1.1$ and $k_1 = k_2 = 0$.

model between the two objective functions previously discussed. When the externality factors are large, one can actually expect that minimizing expenditures can be far from maximizing welfare. Indeed, minimizing expenditures does not account at all for the externalities. Motivated by this observation, we introduce the Intermediate Model (IM), that seeks to minimize the government expenditures minus externalities:

$$IM = (r - k)' \min\{d, q\}. \quad (17)$$

In other words, this model resembles maximizing social welfare without the firms' profits and consumer surplus. Note that when the externalities are zero, this model actually coincides with minimizing expenditures. When externalities are strictly positive, one can see this objective as a modified cost, that accounts for the externalities. Now, one can expect that this objective may be closer to the social welfare as it accounts for externalities. In addition, the firms' profits are actually already being maximized in the second stage and the consumers are offered subsidies from the government, therefore these two terms (suppliers profit and consumer surplus) are somewhat implicitly optimized.

Using the Intermediate Model we just discussed, the government faces the following problem:

$$\begin{aligned} \min_r \mathbb{E}[IM] \\ \mathbb{E}[e' \min\{q(r), d\}] \geq \Gamma \end{aligned} \quad (18)$$

Note that the Intermediate Model is equivalent to minimizing expenditures, when externalities are not very large, since one can show that the optimal solution is obtained when the adoption

constraint is exactly met. However, if the externalities are large, it induces a higher optimal rebate that does not satisfy the adoption constraint with equality.

As before, we characterize the ratios for the social welfare and the government cost relative to the optimal values. The results are presented in the following proposition.

PROPOSITION 5. 1. *Consider a market with large externalities and assume a monopolistic firm with deterministic demand. Then, we have:*

$$\begin{aligned}
 1 - \frac{9(2 + \bar{\gamma})^2}{16(4 + 2\underline{\gamma} + 2\bar{\gamma} + \bar{\gamma}^2)} &\leq \frac{SW_{IM}^N}{SW_{SW}^N} \leq 1 \\
 M &< \frac{GC_{IM}^N}{GC_{GC}^N} \quad \forall M > 0 \\
 16 &\leq \frac{GC_{SW}^N}{GC_{IM}^N} \quad \text{for any instance with large externalities.}
 \end{aligned} \tag{19}$$

Moreover, the bound is asymptotically tight, for $n = 2$.

2. *Consider a competitive setting with deterministic demand. Then, we have:*

$$\begin{aligned}
 1 - \frac{9(\underline{\gamma} + 2)(\bar{\gamma} + 2)(\sqrt{3 + 2\bar{\gamma}} - \sqrt{3 + 2\underline{\gamma}})^2}{16(\sqrt{3 + 2\underline{\gamma}}(\bar{\gamma} + 1) - \sqrt{3 + 2\bar{\gamma}}(\underline{\gamma} + 1))^2} &\leq \frac{SW_{IM}^W}{SW_{SW}^W} \leq 1 \\
 M &< \frac{GC_{SW}^W}{GC_{GC}^W} \quad \forall M > 0 \\
 4 &\leq \frac{GC_{SW}^W}{GC_{IM}^W} \quad \text{for any instance with large externalities.}
 \end{aligned} \tag{20}$$

Proof. See Appendix E. \square

By comparing the ratios from Propositions 4 and 5, one can see that the IM model can be significantly better than the GC model in terms of social welfare. In particular, if the government minimizes expenditures, the loss in welfare can be arbitrarily large when externalities become large. In contrast, the ratio for the IM model achieves a constant guarantee that depends on the externality factors (with a worst case guarantee of 0.25).

In addition, by considering the IM model instead of maximizing social welfare, the government can potentially reduce the cost of the subsidy program significantly. As we previously mentioned, when the government maximizes social welfare, the expenditures can become very large and as a result, the subsidy program is very costly. The results in Proposition 5 show that by considering the IM model, the government can reduce its expenditures by a factor of at least 16 or 4 (in the monopolistic and competitive settings respectively). Consequently, when externalities are large, the best government objective is the IM model as it allows cost reduction while achieving a good social welfare performance.

Note that when $\underline{\gamma} = \bar{\gamma}$, then the social welfare ratio is at most $\frac{7}{16} = 0.4375$. In addition, the worst case is obtained when $\bar{\gamma} = 2$ and $\underline{\gamma} = 0$, with a ratio of 0.25. As noted before, in most practical instances the resulting ratios are close to one so a good social welfare performance is obtained. Note that these bounds are clearly better than the case when the government minimizes expenditures.

Figure 2 presents the values of the ratios in the competitive environment for a particular instance with large externalities as a function of the standard deviation. One can see that both ratios improve (get closer to one) as the standard deviation increases. Nevertheless, the government cost ratio gets closer to one much slower than the social welfare ratio. In addition, for small standard deviations, using the IM model guarantees near optimal social welfare (in this case, the social welfare ratio is between 0.74 and 0.77) while reducing the budget significantly (in this case, by a factor of at least 52).

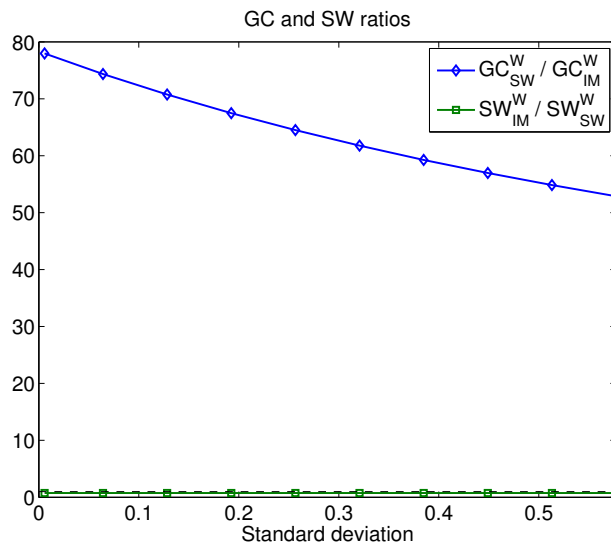


Figure 2 $\bar{d}_1 = 11.52$, $\bar{d}_2 = 5.59$, $B_{11} = 2.76$, $B_{12} = B_{21} = -0.07$, $B_{22} = 0.7$, $c_1 = 4.17$, $c_2 = 0.23$, $\epsilon_1, \epsilon_2 \sim U$, $\Gamma = 3$, $k_1 = 0.83$ and $k_2 = 8.92$.

Figure 3 depicts the lower bounds on the social welfare ratios from Propositions 4 and 5 for the case $\underline{\gamma} = \bar{\gamma} = \gamma$, under a monopolistic setting. Note that when $\gamma \leq 2$, the optimal solution of problem (18) is obtained when the adoption constraint is tight and as result, the social welfare ratio is equal to 1. For $\gamma > 2$, the social welfare ratio for minimizing government cost decreases to zero as the externalities increase, whereas the ratio for the intermediate model remains constant. Consequently, for large externalities one should use the IM model over minimizing the government cost.

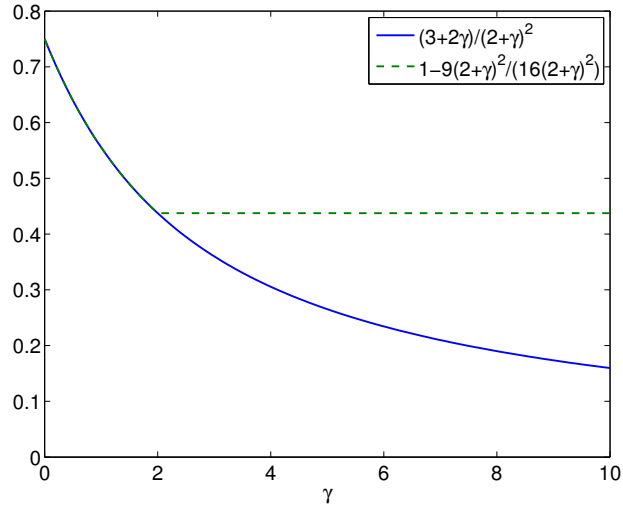


Figure 3 Lower bounds on the social welfare ratios as a function of externality factor γ .

4. Effects of competition

In this section, we examine the effects of competition by comparing the outcomes for the competitive environment relative to the monopolistic setting. Our main goal is to study how competition affects the prices, rebates, effective prices and production quantities. In addition, we are interested in quantifying the impact of competition on the various players involved (government, suppliers and consumers) as well as studying the role of externalities and demand uncertainty. Since we are considering a general asymmetric competition setting, we study the effects of competition on each different market/product. As we previously explained, there exist two different regimes depending on the magnitude of the externalities. Therefore, we divide the analysis into two different cases: small and large externalities. For small externalities, we refer to problem (18) where the optimal solution is obtained when the adoption constraint is exactly met and all the government objectives yield the same optimal rebate. The regime with large externalities refers to the case for which the adoption constraint is not tight when solving problem (18).

We first consider the general case which we call “asymmetric” markets or suppliers, and then the case when the suppliers are symmetric (namely, $\bar{d}_i = \bar{d}_j \forall i, j$, $c_i = c_j \forall i, j$, $B_{ii} = B_{jj} \forall i, j$, $B_{ij} = B_{kl}$ for $\forall i, j, k, l$ $i \neq j, k \neq l$ and $k_i = k_j \forall i, j$).

4.1. Small externalities

4.1.1. Asymmetric case We focus on the case where the externalities are relatively small and the suppliers are asymmetric. Such asymmetries may arise due to different reasons, such as: (i) for some of the products consumers may be more price sensitive than others (e.g., towards luxury versus towards cheap cars); (ii) cross elasticities may be different among two different pairs

of products (degree of substitution); (iii) the marginal costs may be different; (iv) differences in the market shares (e.g., large popular manufacturer versus small new entrant) or (v) differences in the externality factors (difference in gas emissions between different EVs). The following proposition summarizes the comparisons for the optimal variables in the competitive environment relative to the monopolistic setting.⁴

PROPOSITION 6. *Consider n asymmetric suppliers. Then, by comparing the monopolistic setting to the competitive environment, we have:*

$$\begin{aligned} p^N &\geq p^W \\ p^N - r^N &\geq_1 p^W - r^W \\ r^N &\geq r^W \\ q^N &\geq_1 q^W \\ \Pi^N &\geq \Pi^W \\ GC^N &\geq GC^W \end{aligned}$$

Proof. See Appendix F. \square

In this case, the prices and rebates are smaller in the presence of competition. However, not necessarily all the effective prices paid by the consumers are smaller. Instead, we can only say that for at least one product this actually happens. This result differs from the classical insight that competition benefits consumers. In our problem, some of the consumers pay a higher price under the competitive setting. Similarly for production quantities, one can see that at least one product is under produced in the presence of competition. In addition, competition benefits the government in terms of expected cost at the expense of hurting the suppliers in terms of total expected profits. As expected, competition hurts the suppliers. The government, that is leading the game may take advantage of this effect by reducing the rebate offered to consumers while still achieving the desired target adoption level. Consequently, the government can take advantage of competition at the expense of the suppliers. However, the effect on the consumers is not straightforward as they receive lower rebates but also pay a smaller price. In order to draw additional insights on the consumers, we study a simple scenario with deterministic demand and symmetric price elasticities. In this case, one can characterize which segments of consumers benefit from competition.

PROPOSITION 7. *In a deterministic setting with symmetric markets, the competition does not affect the effective prices, quantities, consumer surplus and social welfare. As a result, competition only benefits the government at the expense of hurting the suppliers without affecting the consumers at all. In an asymmetric setting, competition benefits the consumer segment with:*

⁴ We define the relation operator $x \geq_1 y$ ($x \leq_1 y$), such that for any $x, y \in \mathbb{R}^n$, there exists $k \in \{1, \dots, n\}$ that satisfies $x_k \geq y_k$ ($x_k \leq y_k$).

- *The highest marginal cost.*
- *The lowest market share.*

In addition, the net change on effective prices due to the presence of competition is equal to zero, when the asymmetry is on costs and/or market share and not on price elasticities. In other words,

$$\sum_{i=1}^n (p_i^N - r^N) = \sum_{i=1}^n (p_i^W - r^W). \quad (21)$$

Proof. See Appendix G. \square

Recall that as expected, the presence of competition induces lower prices. The government anticipates this effect and sets a lower rebate such that the effective price remains unchanged. When demand is deterministic, produced (or equivalently, sold) units also remain unchanged and therefore, the expected adoption target is still exactly attained. Since the effective price and production quantities are not affected by the presence of competition, consumers surplus and externalities will not be affected either. Interestingly, the social welfare factor remains unchanged too. Indeed, the social welfare stays the same as the welfare from competition is simply transferred from the suppliers to the government. In particular, the price increase is exactly compensated by the rebate reduction.

The second part of Proposition 7 suggests that competition benefits⁵ the segment with the highest marginal cost and/or with the lowest market share. In particular, in a competitive environment, each supplier decides price and production separately, leading to underproduction of the low cost product and overproduction the high cost one. As a result, since the rebate is uniform across the products, the effective price of the low cost product is higher, while the effective price of the high cost product is lower. Similarly, for a product with a large market share (captured in our model by the term \bar{d}_i), a competitive setting leads firms to overproduce the low market share product and underproduce the high market share one. Therefore, the product with low market share benefits from competition as the effective price decreases.

4.1.2. Symmetric case In this section, we focus on the case where the parameters are symmetric across all the products.

Note that all the results from Proposition 6 are still valid. In addition, since all the optimal variables are identical, one can see that the effective price and the production quantity are lower under the competitive environment. Finally, one can also show the following result on the expected consumer surplus.

⁵ We say that a segment benefits from the competition, if the effective price is lower and the produced quantities are higher so that the consumers are clearly better off.

COROLLARY 1. *Consider n symmetric suppliers and assume that $e'A \leq \Gamma$. Then, the expected consumer surplus follows the following relation:*

$$CS^N \leq CS^W.$$

Proof. See Appendix H. \square

The assumption $e'A \leq \Gamma$ ensures that the target adoption level set by the government is relevant and cannot be achieved only by a large noise realization. Note that as the cross elasticity parameters B_{ij} $i \neq j$ approach 0, the markets (and suppliers) become less dependent and as expected, the effect of competition vanishes. In particular, when $B_{ij} = 0 \forall i \neq j$ the monopolistic and competitive settings coincide.

We next consider varying the magnitude of the demand uncertainty, captured by the additive noises ϵ_i . By reducing the demand uncertainty, one can see that the gap in the effective prices decreases. In other words, as A_i goes to 0, $p^W - r^W$ approaches $p^N - r^N$, so that the effect of competition on the effective price diminishes as demand uncertainty decreases. However, the prices and the rebates are still larger for the monopolistic setting. This implies that the effect of competition is totally absorbed by the government when the demand becomes deterministic. More specifically, we still obtain the reduction in the selling price p induced by the competition among the suppliers but in this case, the government can decrease the rebates in a way such that the effective price $p - r$ remains unchanged. This is in contrast to classical insights about competition that suggest that consumers always benefit from competition. The reason of this is because the government aims to achieve a target adoption level. In a competitive environment, the government can anticipate the price reduction due to competition among the suppliers and decrease the rebates so as to achieve the target. Since the sales depend explicitly on the effective price, the price reduction is exactly compensated by the rebate augmentation. As a result, the effective price paid by consumers is not affected by competition in a symmetric and deterministic setting. Interestingly, adding demand uncertainty modifies partially the outcomes. More precisely, the government and the consumers are now sharing the competition benefits at the expense of the suppliers. As before, competition decreases the price, the government reacts by reducing the rebate but in a way that the effective price decreases as well. As a result, the consumers are better-off in a competitive environment, when demand is symmetric and stochastic. Note that the government and the consumers share the benefit of competition. Each player can extract some proportion of the benefits that depends on both the magnitude of the demand uncertainty.

4.2. Large externalities

In this section, we consider the case where externalities are large. More precisely, in that case the externalities are such that the constraint in problem (18) is not tight at optimality. In this setting, the optimal rebate level set by the government is larger than the one that achieves the target adoption constraint with equality. Equivalently, the government offers a larger rebate. This follows from the fact that the externalities generated by an extra dollar of investment are profitable (i.e., they are larger than one dollar). Figure 4 illustrates the marginal government cost and externalities generated by an extra unit of rebate, in an environment where there is competition. The vertical line slightly before 0.5 represents the rebate at which the adoption target constraint is exactly met. One can see that for this rebate value, the marginal externalities are larger than the marginal government cost. Therefore, it is profitable for the government to increase the rebate further and the constraint is not binding at optimality. In other words, the optimal rebate for the intermediate model is attained when the marginal cost equals the marginal externality factor.

As we previously discussed in Section 3, when externalities are large, the government should not aim to minimize expenditures. In particular, the loss in welfare becomes unbounded as the externality factors grow. Therefore, we consider the intermediate model we introduced in (18), where the government minimizes the expenditures corrected by the externality factors. Recall that this objective allows the government to significantly reduce its expenditures, while still achieving a welfare not far from optimal. We next present the results about the impact of competition on the different problem variables.

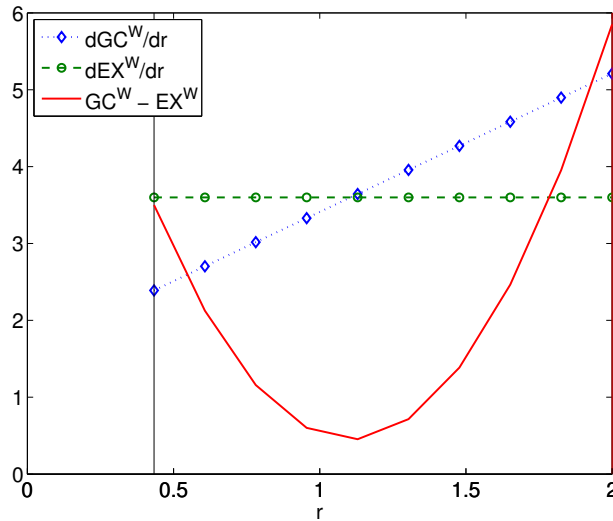


Figure 4 $\bar{d}_1 = 4.0$, $\bar{d}_2 = 3.0$, $B_{11} = B_{22} = 1.0$, $B_{12} = B_{21} = -0.1$, $c_1 = 2.0$, $c_2 = 2.2$, $\epsilon_1, \epsilon_2 = 0$ w.p. 1 and $\Gamma = 2$. The numbers on the y-axis refer to the first two quantities.

PROPOSITION 8. *Consider n asymmetric suppliers with deterministic demand. Then, by comparing the monopolistic setting to the competitive environment, we have:*

$$\begin{aligned} p^N - r^N &\geq_1 p^W - r^W \\ q^N &\leq_1 q^W \\ e'q^N &\leq e'q^W \\ CS^N &\leq CS^W \end{aligned}$$

In the particular case where suppliers are symmetric, we have:

$$\begin{aligned} r^N &= r^W \\ p^N &\geq p^W \\ p^N - r^N &\geq p^W - r^W \\ q^N &\leq q^W \\ \Pi^N &\geq \Pi^W \\ GC^N &\leq GC^W \end{aligned}$$

Proof. See Appendix I. \square

In this case, the rebate level is not necessarily smaller in the competitive environment. Note that under large externalities, competition will induce lower effective prices and larger production on some segments. We present and discuss in detail a concrete example in the next subsection.

Note that the results of Proposition 8 remain valid when demand is stochastic. In particular, all the inequalities remain the same except that in this case, for symmetric suppliers we have: $r^N \geq r^W$.

When externalities are large, Proposition 8 states that in the symmetric case, prices and effective prices will decrease in the presence of competition. Furthermore, quantities produced will increase under competition. A lower effective price will lead to higher demand and as a result, the firms will increase production. Note that this effect did not occur in the case with small externalities. Indeed, since the rebate was set to exactly attain the target adoption level, the firms could produce less units in the presence of competition, even though the effective price was lower. Another interesting result, is that in this case, the total firms' profits and government costs are not necessarily lower in a competitive environment (with asymmetric suppliers), which again, was not the case under small externalities. Actually, for symmetric suppliers the government cost is even higher in the presence of competition.

We next compare the impact of competition on markets with small and large externalities. For the case of symmetric suppliers, one can see that competition always hurts the suppliers in terms

of expected profits no matter how small or large the externalities are. However, the effects on the government and the consumers differ depending on the magnitude of the externalities. On one hand, when externalities are small, the government always benefits from competition in terms of expected cost. On the other hand, when externalities are large the competition has the opposite effect on the government cost (for symmetric suppliers). This difference is driven by the fact that the target adoption level is exactly met, when externalities are small. As a result, the government can take advantage of the competing suppliers by decreasing the rebates offered to consumers, and hence reducing the expenditures. However, recall that for large externalities, the adoption constraint is not tight anymore. As a result, the competition induces larger production quantities and demand so that the government has to subsidize additional units and hence the overall expenditures increase. This is explained by the fact that the government perceives a significant environmental benefit (through the large externalities) from any dollar invested in consumer subsidies and as a result, aims to let competition increase the production quantities.

By studying the expected consumer surplus, one can see that under small externalities, the presence of competition does not always benefit consumers (for asymmetric suppliers). Indeed, since the adoption constraint is exactly met, the production quantities in the presence of competition will be decreased. Consequently, this affects the consumers that dispose of less available supply and may reduce the expected consumer surplus. However, when externalities are large, the adoption constraint is not tight and therefore, the competing suppliers will produce more so that it always benefits consumers.

For asymmetric suppliers, one can note that the impact of competition on the price depends on the magnitude of the externalities. In particular, when externalities are small, all the prices are lower in the competitive environment but when externalities are large, this is not necessarily the case.

5. Computational experiments

In this section, we calibrate our model with actual data and test our insights computationally. Several car manufacturers have launched EVs in the last three years, yet many of them lack of sufficient historical sales data. Therefore, we have decided to consider only the two major EV models for which richer data is available: Chevy Volt and Nissan Leaf⁶. We assume the marginal cost of producing an EV is equal to 90% of the average manufacturer's suggested retail price (MSRP) over the years 2011, 2012 and 2013. We estimate the price elasticities and the intercept terms of the demand (i.e., the matrix B and the vector \bar{d} respectively), by performing a least squares error

⁶ <http://insideevs.com/monthly-plug-in-sales-scorecard/>

estimation of the sales for the two EVs over the past 3 years. Since the data set is small relative to the number of parameters, we assume that the self elasticities are the same (i.e., $B_{11} = B_{22}$). Note that this also helps avoid over fitting. The adoption target Γ is set to 45 (thousand), which corresponds to the amount of EVs sold in 2013 by the two manufacturers. We obtain the following estimates: $\bar{d}_1 = 110.21$, $\bar{d}_2 = 73.43$, $B_{11} = B_{22} = 3.14$, $B_{12} = B_{21} = -0.81$, $c_1 = 30.73$, $c_2 = 28.8$. Finally, we assume that the additive noises ϵ_1 and ϵ_2 are uniformly distributed with support equal to $\pm 10\%$ of their corresponding market shares \bar{d}_1 and \bar{d}_2 .

The positive externalities of an EV correspond to the reduction in CO₂ emissions throughout their lifetime, converted to US dollars. Following the analysis in Arar (2010), the emission rate per unit of energy amounts to 755 [Kg CO₂ × MWh⁻¹]. In addition, the efficiency of an EV is of the order of 0.155 [KWh × Km⁻¹], whereas the average lifetime vehicle mileage⁷ is about 152,137 [Miles]. Consequently, the total emissions of an EV amounts to 755 [Kg CO₂ × MWh⁻¹] × 0.155 [KWh × Km⁻¹] × 152,137 [Miles] × 1.61 [Km × Miles⁻¹] = 28.66 [Ton CO₂]. In the case of a regular gasoline vehicle, the net emission factor can be taken to be 10.6 [CO₂ × gal⁻¹]. Then, considering an average consumption rate of 24.4 [Miles × gal⁻¹], the lifetime emissions generated by a regular vehicle amount to 10.6 [CO₂ × gal⁻¹] × 152,137 [Miles] / (20.4 [Miles × gal⁻¹]) = 78.9 [Ton CO₂]. As a result, an estimate for an EV gas emission reduction is 78.9 – 28.6 = 50.3 [Ton CO₂]. In order to convert this number to US dollars, we use the value assigned by the United States Environmental Protection Agency (EPA)⁸ to a Ton of CO₂. We consider the average value for the years 2014 – 2033, which is 52.0 [\$ × (Ton CO₂)⁻¹] (in US dollars of 2014). Therefore, the monetary positive externality of an EV is equal to 50.3 [Ton CO₂] × 52.0 [\$ × (Ton CO₂)⁻¹] = \$2,612. Therefore, in our model $k_1 = k_2 = k = \$2,612$.

5.1. Demand Uncertainty

First, we test the sensitivity of our results with respect to the magnitude of the demand uncertainty. In other words, we are interested in studying how the impact of competition is affected by demand uncertainty. We assume that the additive noises for each product i is uniformly distributed, i.e., $U[-A_i, A_i]$. We modify the magnitude of the demand uncertainty by varying A_i for each product, ranging from 0 (i.e., a deterministic setting) to 30% of the market share (i.e., a fairly volatile market). Figure 5 shows the ratios of the Government Cost, Industry Profits and Consumer Surplus for different values of A_i . The left plot is under small externalities (in this case, $k = 2.61$) while the right plot assumes large externalities (in this case, $k = 35$).

⁷ <http://www-nrd.nhtsa.dot.gov/Pubs/809952.pdf>

⁸ <http://www.whitehouse.gov/sites/default/files/omb/assets/inforeg/technical-update-social-cost-of-carbon-for-regulator-impact-analysis.pdf>

From the left plot, one can see that when $A_i = 0$ (i.e., a deterministic setting), competition benefits the government (by reducing the expected cost) and hurts the suppliers in terms of profits. As demand uncertainty increases, the benefit of the government is transferred to the consumers. Note that for low demand uncertainty, competition does not always benefit consumers. Nevertheless, incorporating demand uncertainty favors the consumers that can extract some benefit from the presence of the competition, at the expense of the government who needs to pay a larger cost. From the right plot, one can see that when externalities are large, consumers always benefit from competition (see Proposition 8). Surprisingly, increasing demand uncertainty for large externalities leads to a lower benefit for consumers in the competitive environment.

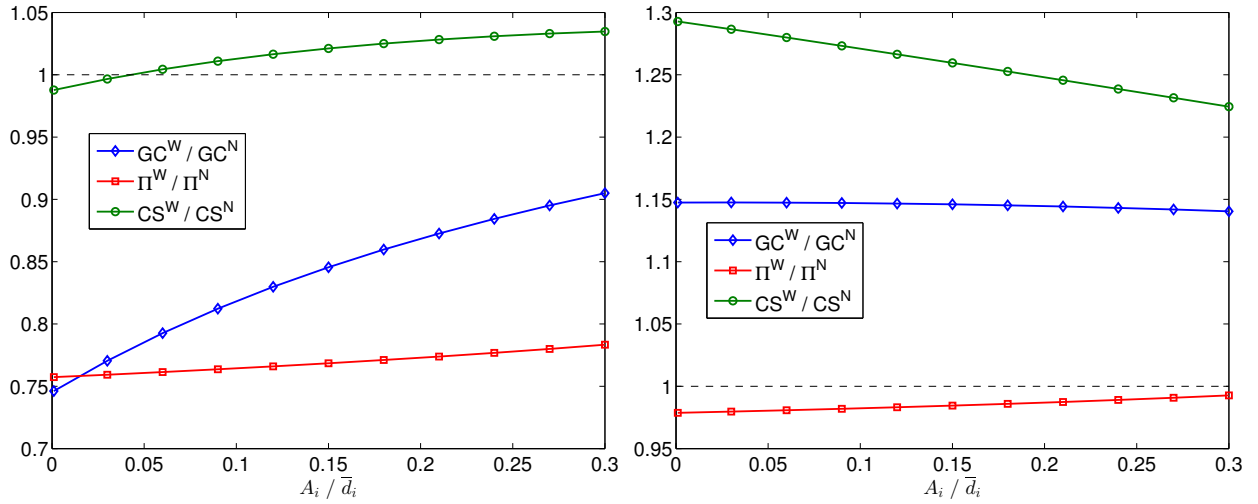


Figure 5 Demand parameters: $\bar{d}_1 = 110.21$, $\bar{d}_2 = 73.43$, $B_{11} = B_{22} = 3.14$, $B_{12} = B_{21} = -0.81$, $c_1 = 30.73$, $c_2 = 28.8$, $\epsilon_1 \sim \bar{d}_1 \times U[-A_1, A_1]$, $\epsilon_2 \sim \bar{d}_2 \times U[-A_2, A_2]$, $\Gamma = 45$. $k = 2.61$ in left plot, and $k = 35$ in right plot.

5.2. Substitution

Next, we analyze the effects of varying the cross-elasticity term $B_{12} = B_{21}$, that captures the degree of competition among the firms. Note that if $B_{12} = 0$ (i.e., independent suppliers), then the monopolist and competitive settings coincide. In Figure 6, we plot the ratios of the Government Cost, Industry Profits and Consumer Surplus for different values of demand substitution captured by $|B_{12}|/B_{11}$.

When externalities are small (left plot of Figure 6), increasing the degree of competition reduces the Government Cost and the firms' profits. As expected, the more intense the competition is, the better it is for the government and the worse it is for the suppliers. Regarding the consumers, they perceive a very small benefit in this case. This follows from the fact that the government can take advantage by counterbalancing the reduction in prices by decreasing the rebates. As a

result, consumers do not benefit much of competition. In the case of large externalities, (see right plot of Figure 6), the suppliers are still hurt from competition as expected. However, the effect of competition on the government and the consumers is quite different than before. In this case, since the target adoption constraint is not tight, the competition induces larger total production (see Proposition 8). As a result, the competition increases the Government Cost and the Consumer Surplus. Figure 6 shows how this effect scales with the degree of substitution. One can see that the impact of competition on the different players (government, suppliers and consumers) highly depends on the magnitude of the externalities.

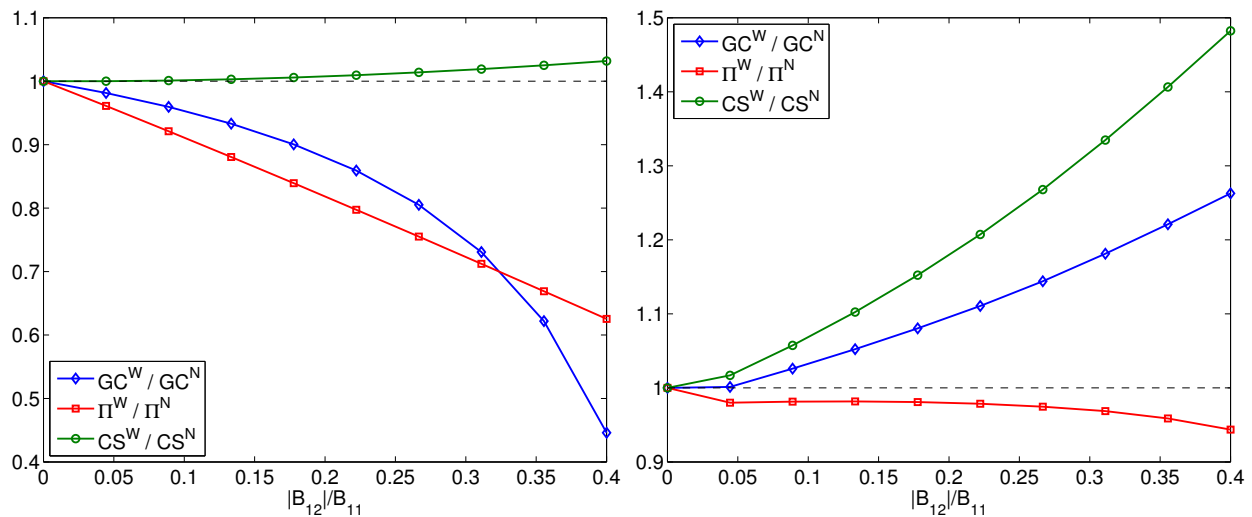


Figure 6 Demand parameters: $\bar{d}_1 = 110.21$, $\bar{d}_2 = 73.43$, $B_{11} = B_{22} = 3.14$, $c_1 = 30.73$, $c_2 = 28.8$, $\epsilon_1 \sim \bar{d}_1 \times U[-0.1, 0.1]$, $\epsilon_2 \sim \bar{d}_2 \times U[-0.1, 0.1]$, $\Gamma = 45$. $k = 2.61$ in left plot, and $k = 30$ in right plot.

5.3. Externalities

As we explained before, the impact of competition on the different players highly depends on the magnitude of the externalities. Next, we study the effect of varying the level of externalities on the Government Cost, Profits of the suppliers and Consumer Surplus by considering a competitive setting. We vary the level of externalities k by changing the price of the carbon. As we previously mentioned, the average value for the years 2014–2033 is 52.0 [$\$ \times (\text{Ton CO}_2)^{-1}$]. The plot in Figure 7 examines the effect on the various players if the price of carbon were to increase in the future. Alternatively, this is equivalent to the case in which technological progress allows significantly reduction of the CO₂ emissions of EVs.

When externalities are small and below a certain threshold, one can see that the price of carbon does not affect either of the quantities of interest. However, when externalities become large (in

this case, when the price of carbon is greater than 520), all three quantities increase with respect to the price of carbon. Consequently, increasing the price of carbon hurts the government but benefits both the suppliers and the consumers. Recall that the government incorporates the environmental benefit of the externalities directly in his objective. As a result, it becomes advantageous for the government to increase rebates. In particular, for any subsidized dollar, the government perceives a great return due to the externalities and to the high price of carbon. Ultimately, it costs more to the government but benefits the other parties. This analysis suggests that for markets with large externalities, the government should be careful about incorporating externalities in his objective, as the cost can become very high, as we show in Section 3.

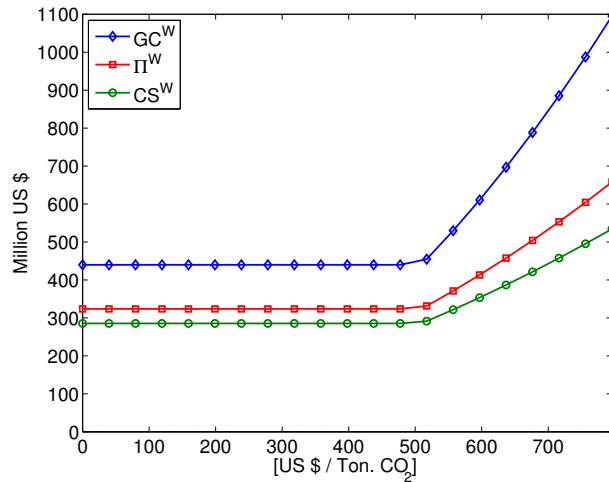


Figure 7 Demand parameters: $\bar{d}_1 = 110.21$, $\bar{d}_2 = 73.43$, $B_{11} = B_{22} = 3.14$, $B_{12} = B_{21} = -0.81$, $c_1 = 30.73$, $c_2 = 28.8$, $\epsilon_1 \sim \bar{d}_1 \times U[-0.1, 0.1]$, $\epsilon_2 \sim \bar{d}_2 \times U[-0.1, 0.1]$, $\Gamma = 45$. The value of the x-axis is the price of a Ton. of CO₂.

6. Conclusions

In this paper we introduce and study a model for competition in green technology adoption. In particular, as it became prevalent in the EV industry, the government offers consumer subsidies with the goal to reach a target adoption level. Multiple competing suppliers then decide price and production so as to maximize expected profits. We compare the outcomes of a competitive environment relative to a monopolistic setting.

We first show that when the government minimizes expenditures, the resulting social welfare remains close to optimal, especially when externality factors are low. As a result, when externalities are not significant, it seems better for the government to minimize the cost of the subsidy program as it provides good social welfare while keeping a reasonable budget. For markets with large

externalities, we first show that minimizing expenditures can yield a very low welfare relative to optimal. We then introduce an intermediate model where the government minimizes expenditures corrected by externality factors and derive tight bounds that characterize the worst case guarantee on the social welfare.

Second, we investigate the impact of competition on the government, the suppliers and the consumers. When the externalities are low, we show that competition induces lower prices and rebates, but not necessarily lower effective prices. For symmetric suppliers, we show that both the effective prices and the production quantities are lower in a competitive environment. As a result, competition hurts the suppliers (that perceive a lower expected profit), benefits the government (that can decrease the rebates) and benefits consumers (that enjoy a larger expected surplus). More precisely, the benefit of competition is shared between the government and the consumers, as a function of the demand uncertainty. When demand is deterministic, the government absorbs the entire benefit and the consumers are not affected by the competition at all. When demand becomes more uncertain, the consumers manage to extract some of the competition benefit and share it with the government.

For markets with large externalities, the impact of competition differs. It becomes optimal for the government to offer subsidies such that the expected sales exceed the target adoption level in order to take advantage of the high environmental benefits. We show that it is not clear anymore that the rebates are lower in a competitive environment and as a result, the government can be hurt by competition. In addition, the consumers will now always benefit from the presence of the competition.

In conclusion, the effect of competition on the different agents depends on the interplay of suppliers' asymmetry, externality factors and demand uncertainty. In most cases, competition tends to hurt the suppliers as expected. However, the impact on the government and the consumers appears to be more subtle, as we illustrate in this paper.

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A. Proof of Proposition 1

Note that we have:

$$\begin{aligned}\nabla_p \pi(p) &= \bar{d} - B(p - er) + \Psi(p) - B(p - c) \\ \nabla_p^2 \pi(p) &= -2B + J_p \Psi(p)\end{aligned}$$

where $[J_p \Psi(p)]_{ij} = \frac{c_i^2}{f_i(F_i^{-1}(1 - \frac{c_i}{p_i}))p_i^3} \mathbb{1}_{\{i=j\}}$. For existence, note that from $\nabla_p \pi = 0$, we obtain $p = \frac{1}{2}B^{-1}\Psi(p) + \frac{1}{2}B^{-1}(\bar{d} + Ber + Bc)$. Starting with $p^{(0)} = c$, then evaluating, we have $p^{(1)} = \frac{1}{2}B^{-1}\Psi(c) + \frac{1}{2}B^{-1}(\bar{d} + Ber + Bc) = \frac{1}{2}B^{-1}(\bar{d} - Bc - A + er) + c \geq c$, so that $p^{(1)} \geq p^{(0)}$. Assume that $p^{(i)} \geq p^{(i-1)}$. We have: $p^{(i+1)} = \frac{1}{2}B^{-1}\Psi(p^{(i-1)}) + \frac{1}{2}B^{-1}(\bar{d} + Ber + Bc) + \frac{1}{2}B^{-1}(\Psi(p^{(i)}) - \Psi(p^{(i-1)})) \geq p^{(i)}$. Note that the sequence is bounded since $p^{(i)} < \frac{1}{2}B^{-1}(\bar{d} + Ber + Bc) \forall i$ and therefore converges. To show uniqueness, a sufficient condition is that $\nabla_p^2 \pi$ is negative definite, which is implied if $-\nabla_p^2 \pi$ is a strictly diagonally dominant (SDD) M-Matrix, i.e., if $\forall i, \forall p_i \geq c_i \frac{c_i^2}{2p_i^3 f_i(F_i^{-1}(1 - \frac{c_i}{p_i}))} \leq e'_i B e$. Since f_i is IFR (i.e., $\frac{d}{dx} \left(\frac{f_i(x)}{1 - F_i(x)} \right) \geq 0$), this is satisfied if $\frac{1}{f_i(A_i)} < 2c_i e'_i B e$. Note that the latter follows directly from Assumption 2. Therefore, we conclude that there exists a unique optimal solution.

B. Proof of Proposition 2

Note that $\mathbb{E}[GC(r)] = r\mathbb{E}[e' \min\{q, d\}] = r\frac{1}{2}e'(\bar{d} + Ber - Bc + \Psi(p(r)))$. We next show that $\mathbb{E}[e' \min\{q, d\}]$ is non-decreasing in r , and so is $\mathbb{E}[GC(r)]$. We have:

$$\frac{d}{dr} \mathbb{E}[e' \min\{q, d\}] = \frac{1}{2}e' B e + \frac{1}{2}e' J_p \Psi(p) \nabla_r p(r).$$

If $\nabla_r p(r) \geq 0$, then the result follows. Taking the derivative with respect to r of $\nabla_p \pi = 0$, we obtain:

$$\begin{aligned}0 &= \nabla_r \nabla_p \pi(p(r), r) = \nabla_r (\Psi(p) + \bar{d} - B(p - er) - B(p - c)) \\ &= J_p \Psi(p) \nabla_r p(r) - 2B \nabla_r p(r) + B e = \nabla_p^2 \pi(p(r)) \nabla_r p(r) + B e\end{aligned}$$

Note that $-\nabla_p^2 \pi$ is a non-singular M-Matrix, since it is a Z-Matrix with positive diagonal elements and SDD. Therefore, $(-\nabla_p^2 \pi)^{-1} \geq 0$. So $\nabla_r p(r) = (-\nabla_p^2 \pi)^{-1} B e \geq 0$ and this concludes the proof.

C. Proof of Proposition 3

Equivalently to the n optimization problems that each firm faces in (13), one can reduce this system to a single optimization problem with $p \in \mathbb{R}^n$ as decision variables and $\pi^W(p) = \pi(p) + \frac{1}{2}(p - c)'(B - D)(p - c)$ as the objective function. Therefore:

$$\begin{aligned}\nabla_p \pi(p)^W &= \bar{d} - B(p - er) + \Psi(p) - D(p - c) \\ \nabla_p^2 \pi(p)^W &= -B - D + J_p \Psi(p)\end{aligned}$$

For existence, note that from $\nabla_p \pi^W = 0$, we have $p = X\Psi(p) + X(\bar{d} + Ber + Bc)$. Starting with $p^{(0)} = c$, then evaluating, we obtain $p^{(1)} = X\Psi(c) + X(\bar{d} + Ber + Bc) = X(\bar{d} - Bc - A + er) + c \geq c$, so that $p^{(1)} \geq p^{(0)}$. Assume that $p^{(i)} \geq p^{(i-1)}$. We have: $p^{(i+1)} = X\Psi(p^{(i-1)}) + X(\bar{d} + Ber + Bc) + X(\Psi(p^{(i)} - \Psi(p^{(i-1)}))) \geq p^{(i)}$. Note the sequence is bounded, since $p^{(i)} < X(\bar{d} + Ber + Bc) \forall i$ and therefore converges. For uniqueness, a sufficient condition is that $\nabla_p^2 \pi^W$ is negative definite, which is implied if $-\nabla_p^2 \pi^W$ is an SDD M-Matrix, i.e., if $\forall i, \forall p_i \geq c_i \frac{c_i^2}{p_i^3 f_i(F_i^{-1}(1 - \frac{c_i}{p_i}))} \leq e'_i(B + D)e$. Since f_i is IFR (i.e., $\frac{d}{dx} \left(\frac{f_i(x)}{1 - F_i(x)} \right) \geq 0$), this is satisfied if $\frac{1}{f_i(A_i)} < c_i e'_i(B + D)e$. Note that the latter follows directly from Assumption 2. Therefore, we conclude that there exists a unique optimal solution.

D. Proof of Proposition 4

1. Let us call q_0^N the vector of quantities produced when there is no rebate in the no competition case. It can be seen that if $e'q_0^N < 2e'q_0^N + e'Bk \leq \Gamma$, both optimization problems (11) and (12) yield the same outcome, since the adoption constraint is tight at optimality in both problems. We next consider the case where $e'q_0^N < \Gamma < 2e'q_0^N + e'Bk$ for which the solution of problem (12) is not tight. The government costs ratio is given by:

$$\begin{aligned} \frac{GC_{SW}^N}{GC_{GC}^N} &= \frac{4(e'q_0^N + e'Bk)(2e'q_0^N + e'Bk)}{2e'Be} \times \left(\frac{2(\Gamma - \frac{e'q_0^N}{2})^2 - \frac{e'q_0^N e'q_0^N}{2}}{e'Be} \right)^{-1} \\ &= \frac{4(e'q_0^N + e'Bk)(2e'q_0^N + e'Bk)}{4(\Gamma - \frac{e'q_0^N}{2})^2 - e'q_0^N e'q_0^N}. \end{aligned}$$

Then making $\Gamma \downarrow e'q_0^N$, the denominator goes to zero, whereas the numerator stays bounded away from zero, since $e'q_0^N < 2e'q_0^N + e'Bk \Rightarrow e'q_0^N + e'Bk > 0$. Then, we obtain:

$$M < \frac{GC_{SW}^N}{GC_{GC}^N} \quad \forall M > 0.$$

We next show that:

$$\frac{3 + 2\underline{\gamma}}{4 + 2\underline{\gamma} + 2\bar{\gamma} + \bar{\gamma}^2} \leq \frac{SW_{GC}^N}{SW_{SW}^N} \leq 1.$$

The second inequality follows from the fact that the government maximizes social welfare. In order to show the first inequality, one can see that:

$$\begin{aligned} \frac{SW_{GC}^N}{SW_{SW}^N} &= \frac{\frac{3}{2}q_0^{N'} B^{-1}q_0^N + k'q_0^N + \frac{(\Gamma - e'q_0^N)(3e'q_0^N + 2e'Bk - \Gamma)}{2e'Be}}{\frac{3}{2}q_0^{N'} B^{-1}q_0^N + k'q_0^N + \frac{(e'q_0^N + e'Bk)^2}{2e'Be}} \geq \frac{\frac{3}{2}q_0^{N'} B^{-1}q_0^N + k'q_0^N}{\frac{3}{2}q_0^{N'} B^{-1}q_0^N + k'q_0^N + \frac{(e'q_0^N + e'Bk)^2}{2e'Be}} \\ &\geq \frac{\frac{3}{2}q_0^{N'} B^{-1}q_0^N + k'q_0^N}{\frac{3}{2}q_0^{N'} B^{-1}q_0^N + k'q_0^N + \frac{(1+\bar{\gamma})^2(e'q_0^N)^2}{2e'Be}} \geq \frac{(\frac{3}{2} + \underline{\gamma})q_0^{N'} B^{-1}q_0^N}{(\frac{3}{2} + \underline{\gamma})q_0^{N'} B^{-1}q_0^N + \frac{(1+\bar{\gamma})^2(e'q_0^N)^2}{2e'Be}} \\ &= \frac{(3 + 2\underline{\gamma})q_0^{N'} B^{-1}q_0^N e'Be}{(3 + 2\underline{\gamma})q_0^{N'} B^{-1}q_0^N e'Be + (1 + \bar{\gamma})^2(e'q_0^N)^2} \end{aligned}$$

$$\begin{aligned}
&= \frac{(3+2\underline{\gamma})q_0^{N'} B^{-1} q_0^N e' B e + (1+\bar{\gamma})^2 (e' q_0^N)^2}{(3+2\underline{\gamma})q_0^{N'} B^{-1} q_0^N e' B e + (1+\bar{\gamma})^2 (e' q_0^N)^2} - \frac{(1+\bar{\gamma})^2 (e' q_0^N)^2}{(3+2\underline{\gamma})q_0^{N'} B^{-1} q_0^N e' B e + (1+\bar{\gamma})^2 (e' q_0^N)^2} \\
&= 1 - \frac{(1+\bar{\gamma})^2 (e' q_0^N)^2}{(3+2\underline{\gamma})q_0^{N'} B^{-1} q_0^N e' B e + (1+\bar{\gamma})^2 (e' q_0^N)^2} \\
&= 1 - \frac{(1+\bar{\gamma})^2 (e' q_0^N)^2}{(3+2\underline{\gamma})(q_0^{N'} B^{-1} q_0^N e' B e - e' q_0^N e' q_0^N) + (4+2\underline{\gamma}+2\bar{\gamma}+\bar{\gamma}^2)(e' q_0^N)^2} \\
&\geq 1 - \frac{(1+\bar{\gamma})^2 (e' q_0^N)^2}{(4+2\underline{\gamma}+2\bar{\gamma}+\bar{\gamma}^2)(e' q_0^N)^2} = 1 - \frac{(1+\bar{\gamma})^2}{4+2\underline{\gamma}+2\bar{\gamma}+\bar{\gamma}^2} = \frac{3+2\underline{\gamma}}{4+2\underline{\gamma}+2\bar{\gamma}+\bar{\gamma}^2}
\end{aligned}$$

The first inequality follows from $e' q_0^N \leq \Gamma \leq 2e' q_0^N + e' B k$ and the second inequality from $D_\gamma B e \leq \bar{\gamma} B e \Rightarrow B^{-1} D_\gamma B e \leq \bar{\gamma} e \Rightarrow q_0^{N'} B^{-1} D_\gamma B e \leq \bar{\gamma} q_0^{N'} e$. The third inequality is obtained from $\underline{\gamma} q_0^{N'} B^{-1} q_0^N \leq k' q_0^N$ and $\frac{d}{dx} \left(\frac{1+x}{1+x+a} \right) = \frac{a}{(1+x+a)^2} \geq 0$ if $a \geq 0$. Finally, the fourth inequality follows from Lemma 2 (see below).

However, one can show that if $k = \gamma(p_0^N - c) = \gamma B^{-1} q_0^N$, for $\gamma \in \mathbb{R}^+$, the gap is tight. The social welfare ratio can be written as:

$$\begin{aligned}
\frac{SW_{GC}^N}{SW_{SW}^N} &= \frac{\frac{3}{2} q_0^{N'} B^{-1} q_0^N + k' q_0^N + \frac{(\Gamma - e' q_0^N)(3e' q_0 - \Gamma)}{2e' B e}}{\frac{3}{2} q_0^{N'} B^{-1} q_0^N + k' q_0^N + \frac{(e' q_0^N + e' B k)^2}{2e' B e}} \\
&\geq \frac{\frac{3}{2} q_0^{N'} B^{-1} q_0^N + k' q_0^N}{\frac{3}{2} q_0^{N'} B^{-1} q_0^N + k' q_0^N + \frac{(e' q_0^N + e' B k)^2}{2e' B e}} \quad (\Gamma \geq e' q_0^N) \\
&= \frac{\frac{3}{2} q_0^{N'} B^{-1} q_0^N + k' q_0^N}{\frac{3}{2} q_0^{N'} B^{-1} q_0^N + k' q_0^N + \frac{(1+\gamma)^2 (e' q_0^N)^2}{2e' B e}} \quad (e' B k = \gamma e' q_0^N) \\
&= \frac{(\frac{3}{2} + \gamma) q_0^{N'} B^{-1} q_0^N}{(\frac{3}{2} + \gamma) q_0^{N'} B^{-1} q_0^N + \frac{(1+\gamma)^2 (e' q_0^N)^2}{e' B e}} \quad (q_0^{N'} k = \gamma q_0^{N'} B^{-1} q_0^N) \\
&= \frac{(3+2\gamma) q_0^{N'} B^{-1} q_0^N e' B e}{(3+2\gamma) q_0^{N'} B^{-1} q_0^N e' B e + (1+\gamma)^2 (e' q_0^N)^2} \\
&= \frac{(3+2\gamma) q_0^{N'} B^{-1} q_0^N e' B e}{(3+2\gamma)(q_0^{N'} B^{-1} q_0^N e' B e - e' q_0^N e' q_0^N) + (2+\gamma)^2 (e' q_0^N)^2} \\
&= \frac{(3+2\gamma) q_0^{N'} B^{-1} q_0^N e' B e + (1+\gamma)^2 (e' q_0^N)^2}{(3+2\gamma)(q_0^{N'} B^{-1} q_0^N e' B e - e' q_0^N e' q_0^N) + (2+\gamma)^2 (e' q_0^N)^2} - \frac{(1+\gamma)^2 (e' q_0^N)^2}{(1+\gamma)^2 (e' q_0^N)^2} \\
&= \frac{(3+2\gamma)(q_0^{N'} B^{-1} q_0^N e' B e - e' q_0^N e' q_0^N) + (2+\gamma)^2 (e' q_0^N)^2}{(1+\gamma)^2 (e' q_0^N)^2} \\
&= 1 - \frac{(3+2\gamma)(q_0^{N'} B^{-1} q_0^N e' B e - e' q_0^N e' q_0^N) + (2+\gamma)^2 (e' q_0^N)^2}{(3+2\gamma)(q_0^{N'} B^{-1} q_0^N e' B e - e' q_0^N e' q_0^N) + (2+\gamma)^2 (e' q_0^N)^2} \\
&\geq 1 - \frac{(1+\gamma)^2 (e' q_0^N)^2}{(2+\gamma)^2 (e' q_0^N)^2} \quad (q_0^{N'} B^{-1} q_0^N e' B e - e' q_0^N e' q_0^N \geq 0) \\
&= 1 - \left(\frac{1+\gamma}{2+\gamma} \right)^2 = \frac{3+2\gamma}{(2+\gamma)^2}
\end{aligned}$$

Note that $1 - \left(\frac{1+\gamma}{2+\gamma} \right)^2 \in \left[\frac{5}{9}, \frac{3}{4} \right]$ when $\gamma \in [0, 1]$. In addition, when $\gamma > 0$, then $\frac{SW_{GC}^N}{SW_{SW}^N} \rightarrow 0$ as $\gamma \rightarrow \infty$.

2. Let us call q_0^W the vector of quantities produced when there is no rebate in the case with competition. One can see that if $e'q_0^W < e'q_0^W + \frac{e'DX(\bar{d}-Bc-q_0^W+Bk)}{e'DXBXD_e}e'DXBe < \Gamma$, both optimization problems, the analogous of (11) and (12) in the presence of competition, yield the same outcomes since in both the adoption constraint is tight at optimality. We next consider the case where $e'q_0^W < \Gamma < e'q_0^W + \frac{e'DX(\bar{d}-Bc-q_0^W+Bk)}{e'DXBXD_e}e'DXBe$ for which the solution of problem (12) is not tight. The government costs ratio is given by:

$$\frac{GC_{SW}^W}{GC_{GC}^W} = \left[\frac{e'DX(\bar{d}-Bc-q_0^W+Bk)}{e'(DX)^2Be} \left(e'q_0^W + \frac{e'DX(\bar{d}-Bc-q_0^W+Bk)e'DXBe}{e'(DX)^2Be} \right) \right] \times \frac{e'DXBe}{\Gamma(\Gamma-e'q_0^W)}.$$

If $\Gamma \downarrow e'q_0^W$, then the denominator goes to zero. Note that the left term of the numerator is bounded away from zero if the optimal rebate level when maximizing SW is positive, i.e., if $e'DX(\bar{d}-Bc-q_0^W+Bk) > 0$. We have:

$$\begin{aligned} e'DX(\bar{d}-Bc-q_0^W+Bk) &= e'DX(BX(\bar{d}-Bc)+Bk) \\ &= e'BXDX(\bar{d}-Bc)+e'BXDK \quad \left(I-DX=BX, DXB=BXD \right) \\ &\geq 0. \end{aligned}$$

Note that $e'B$ is positive since B is an M-Matrix and strictly diagonally dominant. XD is positive since D is positive and so is $(B+D)^{-1}$ (as B is diagonally dominant and an M-Matrix), then $e'BXD \geq 0$. In addition, since $2q_0^N = \bar{d}-Bc \geq 0$ and $k \geq 0$, the last inequality is satisfied and we have:

$$M < \frac{GC_{SW}^W}{GC_{GC}^W} \quad \forall M > 0.$$

We next show that:

$$\frac{3+2\bar{\gamma}}{(2+\bar{\gamma})^2} \leq \frac{SW_{GC}^W}{SW_{SW}^W} \leq 1.$$

The second inequality follows from the fact that the government maximizes social welfare. To show the first inequality, one can see that:

$$\begin{aligned} \frac{SW_{GC}^W}{SW_{SW}^W} &= \frac{(\bar{d}-Bc-\frac{1}{2}q_0^W+Bk)'B^{-1}q_0^W + \frac{\Gamma-e'q_0^W}{e'DXBe}e'DX(\bar{d}-Bc-q_0^W+Bk) - \frac{1}{2}\left(\frac{\Gamma-e'q_0^W}{e'DXBe}\right)^2 e'(DX)^2Be}{(\bar{d}-Bc-\frac{1}{2}q_0^W+Bk)'B^{-1}q_0^W + \frac{1}{2}\frac{(e'DX(\bar{d}-Bc-q_0^W+Bk))^2}{e'(DX)^2Be}} \\ &= 1 - \frac{\frac{1}{2e'(DX)^2Be} \left(e'DX(\bar{d}-Bc-q_0^W+Bk) - (\Gamma-e'q_0^W)\frac{e'(DX)^2Be}{e'DXBe} \right)^2}{(\bar{d}-Bc-\frac{1}{2}q_0^W+Bk)'B^{-1}q_0^W + \frac{1}{2}\frac{(e'DX(\bar{d}-Bc-q_0^W+Bk))^2}{e'(DX)^2Be}} \\ &\geq 1 - \frac{\frac{1}{2e'(DX)^2Be} (e'DX(\bar{d}-Bc-q_0^W+Bk))^2}{(\bar{d}-Bc-\frac{1}{2}q_0^W+Bk)'B^{-1}q_0^W + \frac{1}{2}\frac{(e'DX(\bar{d}-Bc-q_0^W+Bk))^2}{e'(DX)^2Be}} \quad \left(\Gamma > e'q_0^W \right) \\ &= 1 - \frac{(e'DX(\bar{d}-Bc-q_0^W+Bk))^2}{2(\bar{d}-Bc-\frac{1}{2}q_0^W+Bk)'B^{-1}q_0^W e'(DX)^2Be + (e'DX(\bar{d}-Bc-q_0^W+Bk))^2} \\ &= 1 - \frac{(e'DX(BD^{-1}+BD_\gamma B^{-1})q_0^W)^2}{2q_0^{W'}(BD^{-1}+\frac{I}{2}+D_\gamma)'B^{-1}q_0^W e'(DX)^2Be + (e'DX(BD^{-1}+BD_\gamma B^{-1})q_0^W)^2} \end{aligned}$$

One can show that there exists a vector $u \in \mathbb{R}_+^n$ such that (the details are not reported due to space limitations):

$$\frac{q_0^{W'}(BD^{-1} + \frac{1}{2}I + D_\gamma)B^{-1}q_0^W}{(e'DX(BD^{-1} + BD_\gamma B^{-1})q_0^W)^2} \geq \frac{u'(\frac{3}{2}I + D_\gamma)B^{-1}u}{(e'DX(I + D_\gamma)u)^2}.$$

Therefore, we obtain:

$$\begin{aligned} \frac{SW_{GC}^W}{SW_{SW}^W} &\geq 1 - \frac{(e'DX(I + D_\gamma)u)^2}{2u'(\frac{3}{2}I + D_\gamma)B^{-1}ue'DXBXDe + (e'DX(I + D_\gamma)u)^2} \\ &\geq 1 - \frac{(e'DX(I + D_\gamma)u)^2}{(e'DX(3I + 2D_\gamma)^{\frac{1}{2}}u)^2 + (e'DX(I + D_\gamma B^{-1})u)^2} \quad (\text{From Lemma 2}) \\ &= \frac{(e'DX(3I + 2D_\gamma)^{\frac{1}{2}}u)^2}{(e'DX(3I + 2D_\gamma)^{\frac{1}{2}}u)^2 + (e'DX(I + D_\gamma)u)^2} \\ &\geq \left(\frac{e'DX(3I + 2D_\gamma)^{\frac{1}{2}}u}{e'DX(2I + D_\gamma)u} \right)^2 \quad (\text{From Lemma 3}) \end{aligned}$$

Let $x_i = [e'DX]_i \cdot u_i \geq 0$. We want to show that:

$$\begin{aligned} \frac{\sum_{i=1}^n x_i \sqrt{3+2\gamma_i}}{\sum_{i=1}^n x_i (2+\gamma_i)} &\geq \frac{\sqrt{3+2\bar{\gamma}}}{2+\bar{\gamma}} \Leftrightarrow (2+\bar{\gamma}) \sum_{i=1}^n x_i \sqrt{3+2\gamma_i} \geq (\sqrt{3+2\bar{\gamma}}) \sum_{i=1}^n x_i (2+\gamma_i) \\ &\Leftrightarrow (2+\bar{\gamma}) \sqrt{3+2\gamma_i} \geq (\sqrt{3+2\bar{\gamma}})(2+\gamma_i) \quad \forall i \in \{1, \dots, n\} \\ &\Leftrightarrow \frac{\sqrt{3+2\gamma_i}}{2+\gamma_i} \geq \frac{\sqrt{3+2\bar{\gamma}}}{2+\bar{\gamma}} \end{aligned}$$

But note that: $\frac{d}{d\gamma_i} \left(\frac{\sqrt{3+2\gamma_i}}{2+\gamma_i} \right) = -\frac{1+\gamma_i}{(2+\gamma_i)^2 \sqrt{3+2\gamma_i}} \leq 0$. As a result, we obtain:

$$\left(\frac{e'DX(3I + 2D_\gamma)^{\frac{1}{2}}u}{e'DX(2I + D_\gamma)u} \right)^2 \geq \frac{3+2\bar{\gamma}}{(2+\bar{\gamma})^2},$$

and this concludes the proof. \square

LEMMA 1. Assume $D, P \in \mathbb{R}^{n \times n}$ such that D is a diagonal positive matrix, P a non-negative symmetric matrix and $a \in \mathbb{R}^n$ such that $a \geq 0$. Then: $a'DPa \geq a'D^{\frac{1}{2}}PD^{\frac{1}{2}}a$.

Proof.

$$\begin{aligned} a'DPa &= \sum_{i=1}^n \sum_{j=1}^n P_{i,j} D_{i,i} D_{j,j} a_i = \sum_{i=1}^n P_{i,i} D_{i,i} D_{i,i} a_i + \sum_{i=1}^n \sum_{j=i+1}^n P_{i,j} D_{i,i} D_{j,j} (a_i + a_j) \\ &\geq \sum_{i=1}^n P_{i,i} D_{i,i} D_{i,i} a_i + \sum_{i=1}^n \sum_{j=i+1}^n P_{i,j} D_{i,i} D_{j,j} 2\sqrt{a_i a_j} \\ &= \sum_{i=1}^n \sum_{j=1}^n P_{i,j} D_{i,i} D_{j,j} \sqrt{a_i} \sqrt{a_j} = a'D^{\frac{1}{2}}PD^{\frac{1}{2}}a \end{aligned}$$

\square

LEMMA 2. Let $y, z \in \mathbb{R}^n$ be non negative and $C \in \mathbb{R}^{n \times n}$ be a symmetric positive-semidefinite matrix. Then, $z' C z y' C^{-1} y - (y' z)^2 \geq 0$.

Proof. Let $x = \sqrt{z' C z} C^{\frac{1}{2}} \left(\frac{C z z'}{z' C z} - I \right) y$.

$$\begin{aligned} v'v &= z' C z y' \left(\frac{z z' C}{z' C z} - I \right) C^{-1} \left(\frac{C z z'}{z' C z} - I \right) y \\ &= z' C z y' \left(\frac{z z' C}{z' C z} - I \right) \left(\frac{z z'}{z' C z} - C^{-1} \right) y = z' C z y' \left(\frac{z z' C z z'}{(z' C z)^2} - \frac{z z'}{z' C z} - \frac{z z'}{z' C z} + C^{-1} \right) y \\ &= z' C z y' \left(\frac{z z'}{z' C z} - \frac{z z'}{z' C z} - \frac{z z'}{z' C z} + C^{-1} \right) y = z' C z y' \left(C^{-1} y - \frac{z z'}{z' C z} \right) y = z' C z y' C^{-1} y - y' z z' y \end{aligned}$$

Note that the positive semi-definiteness ensures a unique square root of C . Nevertheless, this assumption can be relaxed and the Lemma remains valid. \square

LEMMA 3. Assume $y, z \in \mathbb{R}^n$ are non negative vectors and $D \in \mathbb{R}^{n \times n}$ such that D is a diagonal positive matrix. Then, $(z'(3I + 2D)^{1/2}y)^2 + (z'(I + D)y)^2 \leq (z'(2I + D)y)^2$.

Proof.

$$\begin{aligned} &(z'(3I + 2D)^{1/2}y)^2 + (z'(I + D)y)^2 = \\ &= \sum_{i=1}^n \sum_{j=1}^n z_i y_i z_j y_j (1 + D_{i,i})(1 + D_{j,j}) + \sum_{i=1}^n \sum_{j=1}^n z_i y_i z_j y_j \sqrt{(3 + 2D_{i,i})(3 + 2D_{j,j})} \\ &= \sum_{i=1}^n \sum_{j=1}^n z_i y_i z_j y_j (1 + D_{i,i} + D_{j,j} + D_{i,i} D_{j,j} + \sqrt{9 + 6D_{i,i} + 6D_{j,j} + 4D_{i,i} D_{j,j}}) \\ &\leq \sum_{i=1}^n \sum_{j=1}^n z_i y_i z_j y_j (1 + D_{i,i} + D_{j,j} + D_{i,i} D_{j,j} + \sqrt{9 + 6D_{i,i} + 6D_{j,j} + D_{i,i}^2 + 2D_{i,i} D_{j,j} + D_{j,j}^2}) \\ &= \sum_{i=1}^n \sum_{j=1}^n z_i y_i z_j y_j (1 + D_{i,i} + D_{j,j} + D_{i,i} D_{j,j} + \sqrt{(D_{i,i} + D_{j,j} + 3)^2}) \\ &= \sum_{i=1}^n \sum_{j=1}^n z_i y_i z_j y_j (1 + D_{i,i} + D_{j,j} + D_{i,i} D_{j,j} + (D_{i,i} + D_{j,j} + 3)) \\ &= \sum_{i=1}^n \sum_{j=1}^n z_i y_i z_j y_j (4 + 2D_{i,i} + 2D_{j,j} + D_{i,i} D_{j,j}) = (z'(2I + D)y)^2 \end{aligned}$$

\square

E. Proof of Proposition 5

1. If $\frac{2e'q_0^N + k'Be}{4} < \Gamma$ the constraint of Problem (18) is tight, and in this case, we are back to the small externalities case so that Proposition 4 applies. We next consider the case where $e'q_0 < \Gamma \leq \frac{2e'q_0^N + k'Be}{4}$ (i.e., large externalities and the adoption constraint is not tight). Note that $2e'q_0 < k'Be$. Observe that the region of Γ is such that $e'q_0 < \Gamma \leq \frac{2e'q_0^N + k'Be}{4} < 2e'q_0^N + k'Be$, then the optimal solution of Problem (12) is such that the adoption constraint is not tight.

The government cost ratio is given by:

$$\begin{aligned} \frac{GC_{IM}^N}{GC_{GC}^N} &= \frac{(k'Be - 2e'q_0^N)(k'Be + 2e'q_0^N)}{8e'Be} \times \frac{e'Be}{2\Gamma(\Gamma - e'q_0^N)} \\ &= \frac{(k'Be - 2e'q_0^N)(k'Be + 2e'q_0^N)}{16\Gamma(\Gamma - e'q_0^N)}, \end{aligned}$$

where the expression approaches $+\infty$ when $\Gamma \downarrow e'q_0^N$, since $k'Be - 2e'q_0^N = (k'Be + 2e'q_0^N) - 4e'q_0^N > 4\Gamma - 4\Gamma = 0$. Therefore, we have:

$$M < \frac{GC_{IM}^N}{GC_{GC}^N} \quad \forall M > 0.$$

We next show the inequality for the social welfare ratio. In the general case, we have: $k \leq D_\gamma B^{-1} q_0^N$. Note that $\bar{\gamma} \geq 2$, as otherwise, if $\bar{\gamma} < 2$, we have $k'Be = e'BD_\gamma B^{-1} q_0^N \leq \bar{\gamma} e'BB^{-1} q_0^N = \bar{\gamma} e'q_0^N < 2e'q_0^N$ which is a contradiction. The social welfare ratio is given by:

$$\begin{aligned} \frac{SW_{IM}^N}{SW_{SW}^N} &= \left(\frac{3}{2} q_0^{N'} B^{-1} q_0^N + k'q_0^N + \frac{(k'Be - 2e'q_0^N)(7k'Be + 10e'q_0^N)}{32e'Be} \right) \\ &\quad \times \left(\frac{3}{2} q_0^{N'} B^{-1} q_0^N + k'q_0^N + \frac{(k'Be + e'q_0^N)^2}{2e'Be} \right)^{-1} \\ &= 1 - \frac{\frac{(e'q_0^N + k'Be)^2}{2e'Be} - \frac{(k'Be - 2e'q_0^N)(7k'Be + 10e'q_0^N)}{32e'Be}}{\frac{3}{2} q_0^{N'} B^{-1} q_0^N + k'q_0^N + \frac{(k'Be + e'q_0^N)^2}{2e'Be}} \\ &= 1 - \frac{16(e'q_0^N + k'Be)^2 - (k'Be - 2e'q_0^N)(7k'Be + 10e'q_0^N)}{48q_0^{N'} B^{-1} q_0^N e'Be + 32k'q_0^N e'Be + 16(k'Be + e'q_0^N)^2} \\ &= 1 - \frac{9(2e'q_0^N + k'Be)^2}{48q_0^{N'} B^{-1} q_0^N e'Be + 32k'q_0^N e'Be + 16(k'Be + e'q_0^N)^2} \\ &\geq 1 - \frac{9(2e'q_0^N + k'Be)^2}{48q_0^{N'} B^{-1} q_0^N e'Be + 32\underline{\gamma} q_0^{N'} B^{-1} q_0^N e'Be + 16(k'Be + e'q_0^N)^2} \\ &= 1 - \frac{9(2e'q_0^N + k'Be)^2}{16(3 + 2\underline{\gamma})(q_0^{N'} B^{-1} q_0^N e'Be - (e'q_0^N)^2) + 16(k'Be + e'q_0^N)^2 - (3 + 2\underline{\gamma})(e'q_0^N)^2} \\ &\geq 1 - \frac{9(2e'q_0^N + \bar{\gamma}e'q_0^N)^2}{16(3 + 2\underline{\gamma})(q_0^{N'} B^{-1} q_0^N e'Be - (e'q_0^N)^2) + 16(\bar{\gamma}e'q_0^N + e'q_0^N)^2 - (3 + 2\underline{\gamma})(e'q_0^N)^2} \\ &= 1 - \frac{9(2 + \bar{\gamma})^2 (e'q_0^N)^2}{16(3 + 2\underline{\gamma})(q_0^{N'} B^{-1} q_0^N e'Be - (e'q_0^N)^2) + 16(4 + 2\underline{\gamma} + 2\bar{\gamma} + \bar{\gamma}^2)(e'q_0^N)^2} \\ &\geq 1 - \frac{9(2 + \bar{\gamma})^2 (e'q_0^N)^2}{16(4 + 2\underline{\gamma} + 2\bar{\gamma} + \bar{\gamma}^2)(e'q_0^N)^2} \\ &= 1 - \frac{9(2 + \bar{\gamma})^2}{16(4 + 2\underline{\gamma} + 2\bar{\gamma} + \bar{\gamma}^2)} \end{aligned}$$

The first inequality follows from $k'q_0^N \geq \underline{\gamma} q_0^{N'} B^{-1} q_0^N$ and the second inequality from $k'Be \leq \bar{\gamma} e'q_0$. Finally, the third inequality is implied by Lemma 2. Note that $\frac{9(2+\bar{\gamma})^2}{16(4+2\underline{\gamma}+2\bar{\gamma}+\bar{\gamma}^2)}$ is increasing in $\bar{\gamma}$ and $\underline{\gamma}$. Note also that $\bar{\gamma} \geq 2$, $\underline{\gamma} \geq 0$ so that the worst case is obtained when $\bar{\gamma} = 2$ and $\underline{\gamma} = 0$, yielding a ratio of 0.25.

Finally, we show the last inequality for the government cost ratio:

$$\begin{aligned}\frac{GC_{SW}^N}{GC_{IM}^N} &= \frac{2(e'q_0 + e'Bk)(2e'q_0 + e'Bk)}{e'Be} \times \frac{8e'Be}{(k'Be - 2e'q_0)(k'Be + 2e'q_0)} \\ &= 16 \frac{k'Be + e'q_0}{k'Be - 2e'q_0} \geq 16.\end{aligned}$$

2. We next consider the competitive environment. The lower bound for the social welfare ratio can be shown in a similar way than before, and is not reported due to space limitations.

We next show the bounds for the government costs ratio. Comparing the costs for the IM and GC models, we obtain:

$$\begin{aligned}\frac{GC_{IM}^W}{GC_{GC}^W} &= \frac{(k'DXBe - e'q_0^W)(k'DXBe + e'q_0^W)}{4e'DXBe} \times \frac{e'DXBe}{\Gamma(\Gamma - e'q_0^W)} \\ &= \frac{(k'DXBe - 2e'q_0^W)(k'DXBe + e'q_0^W)}{4\Gamma(\Gamma - e'q_0^W)}.\end{aligned}$$

Note that the above expression approaches $+\infty$ when $\Gamma \downarrow e'q_0^W$, since $k'DXBe - 2e'q_0^W > 0$. Therefore:

$$M < \frac{GC_{SW}^W}{GC_{GC}^W} \quad \forall M > 0.$$

We next show the second inequality for the government costs ratio (using the IM model relative to SW):

$$\frac{GC_{SW}^W}{GC_{IM}^W} \geq 4.$$

It is sufficient to show that $\frac{r_{SW}^W}{r_{IM}^W} \geq 2$ and $\frac{e'q_{SW}^W}{e'q_{IM}^W} \geq 2$. We have:

$$\begin{aligned}\frac{r_{SW}^W}{r_{IM}^W} &= \frac{e'DX(\bar{d} - Bc - q_0^W + Bk)}{e'DXBXDe} \times \frac{2e'DXBe}{k'DXBe - e'q_0^W} \\ &= 2 \frac{e'DXBe}{e'DXBXDe} \frac{e'DXBe - e'q_0^W + e'DX(\bar{d} - Bc)}{e'DXBe - e'q_0^W} \\ &\geq 2 \quad \left(\frac{e'DXBe}{e'DXBXDe} \geq 1 \right)\end{aligned}$$

The total production quantities ratio can be written as:

$$\begin{aligned}\frac{e'q_{SW}^W}{e'q_{IM}^W} &= \left(e'q_0^W + e'DX(\bar{d} - Bc - q_0^W + Bk) \frac{e'DXBe}{e'DXBXDe} \right) \times \frac{1}{e'q_0^W + \frac{k'DXBe - e'q_0^W}{2}} \\ &\geq 2 \frac{e'q_0^W + e'DXBk + e'DX(\bar{d} - Bc - q_0^W)}{e'q_0^W + e'DXBk} \quad \left(\frac{e'DXBe}{e'DXBXDe} \geq 1 \right) \\ &= 2 \frac{e'q_0^W + e'DXBk + e'DXBX(\bar{d} - Bc)}{e'q_0^W + e'DXBk} \geq 2 \frac{e'q_0^W + e'DXBk}{e'q_0^W + e'DXBk} = 2\end{aligned}$$

Note that $1 \leq 1 + \frac{e'DXBXBe}{e'DXBXDe} = \frac{e'DXBe}{e'DXBXDe}$ and therefore, $1 \leq \frac{e'DXBe}{e'DXBXDe} \leq 2$. \square

F. Proof of Proposition 6

We divide the proof in several steps.

1. We first show that at least one of the prices for the monopolist setting is larger, i.e., $p^N \geq_1 p^W$. Recall that for each setting (with and without competition), one can write a system of $n + 1$ optimal equations in the $n + 1$ decision variables (p and r). The first n equations are obtained from the first order conditions on the prices and are given by (for more details, see Appendix A):

$$\text{Competition: } \nabla_p \pi(p, z) = \bar{d} - B(p - er) - \Theta(z) - D(p - c)$$

$$\text{Non-Competition: } \nabla_p \pi(p, z) = \bar{d} - B(p - er) - \Theta(z) - B(p - c)$$

The last equation is obtained by optimizing the government problem and is the same in both settings (using the tightness of the adoption constraint):

$$e'(\bar{d} - B(p - r) - \Theta(z)) = \Gamma. \quad (22)$$

By summing up the first n equations (from the n different products) and subtracting equation (22) from this sum, we obtain:

$$\text{Competition: } \sum_{i=1}^n B_{ii} (p_i^W - c_i) = \Gamma \quad (23)$$

$$\text{Non-Competition: } \sum_{i=1}^n \left(B_{ii} - \sum_{j \neq i} B_{ij} \right) (p_i^N - c_i) = \Gamma \quad (24)$$

By comparing the above two equations and using the assumption that the matrix B is strictly diagonal dominant, one can see that at least one of the Non-Competition prices has to be larger, i.e., $p^N \geq_1 p^W$.

2. We next show that all the prices under competition are larger, i.e., $p_i^N \geq p_i^W \forall i = 1, \dots, n$. We present the proof for the case with $n = 2$ in order to simplify the illustration. One can extend the same argument in an iterative fashion for $n > 2$. From the previous step, we know that at least for one product (without loss of generality, product 1), we have $p_1^N \geq p_1^W$. We assume by contradiction that $p_2^N < p_2^W$. We then look at the optimality equation for product 2 in both settings. By comparing the two equations, one can see that we need to require $r^N < r^W$ (the details are omitted due to space limitations). Therefore, we have: $p_1^N - r^N > p_1^W - r^W$. In addition, by still looking at the same equations, one can see that: $p_2^N - r^N > p_2^W - r^W$. We next look at the last equation (the tightness of the adoption constraint). Recall that this equation is the same for both setting. For $n = 2$, we have:

$$\bar{d}_1 + \bar{d}_2 - B_{11}(p_1 - r) - B_{22}(p_2 - r) + B_{12}(p_2 - r) + B_{21}(p_1 - r) - \Theta(z_1) - \Theta(z_2) = \Gamma.$$

We know that: $p_2^N - r^N > p_2^W - r^W$, $r^N < r^W$ and $p_1^N \geq p_1^W$. In addition, since $p_2^N < p_2^W$, we also have $\Theta(z_2^N) > \Theta(z_2^W)$. By using Assumption 2 and the strict diagonal dominance of the matrix B , we obtain a contradiction so that the adoption constraint cannot be achieved in both settings. Therefore, we conclude that $p_1^N \geq p_1^W$ and $p_2^N \geq p_2^W$, or more generally $p_i^N \geq p_i^W \forall i = 1, 2, \dots, n$.

3. We next show that at least one of the effective prices for the monopolist setting is larger, i.e., $p^N - r^N \geq_1 p^W - r^W$. From the previous step, we have $p_i^N \geq p_i^W \forall i = 1, 2, \dots, n$ and therefore $\Theta(z_i^N) \leq \Theta(z_i^W) \forall i = 1, 2, \dots, n$. In order to satisfy the adoption constraint in both settings, it has to be that $p^N - r^N \geq_1 p^W - r^W$.

4. We next show the inequality for the rebates, i.e., $r^N \geq r^W$. By again looking at the adoption target constraint and using the fact that $p_i^N \geq p_i^W \forall i = 1, 2, \dots, n$, one can see that in order to satisfy the constraint in both settings, we need to have $r^N \geq r^W$.

5. We next show the inequality for production quantities, i.e., $q^N \geq_1 q^W$. Note that the sum of all the quantities is given by: $e'q = e'(\bar{d} - B(p - er) + z) = \Gamma + e'(\Theta(z) + z)$. We next show that $e'q$ is non-increasing in each component of the vector z . We have:

$$\frac{d(e'q)}{dz_i} = \int_{z_i}^{A_i} (\epsilon_i - z_i) f(\epsilon_i) d\epsilon_i + \int_{-A_i}^{z_i} z_i f(\epsilon_i) d\epsilon_i + \int_{z_i}^{-A_i} z_i f(\epsilon_i) d\epsilon_i = F_i(z_i) \geq 0.$$

Recall that we have shown that $p^N \geq p^W$ and as a result, $z^N \geq z^W$. Therefore, we have $e'q^N \geq e'q^W$ and consequently, at least one component of q^N must be larger.

6. We next show the inequality for the profits: $\Pi^N \geq \Pi^W$. Consider $\Pi^N(p, z, r)$ and $\Pi^W(p, z, r)$ evaluated at a given value of the rebate r , chosen by the government. We have:

$$\Pi^N(p^N, z^N, r^N) = \max_{p, z} \Pi^N(p, z, r^N) \geq \max_{p, z} \Pi^N(p, z, r^W) \geq \Pi^N(p^W, z^W, r^W).$$

The first equality comes from the definition of p^N and z^N . Then, the first inequality follows from the fact that $r^N \geq r^W$ (shown in step 4). Indeed, we have: $\Pi^N(p + e(r^N - r^W), z, r^N) = (p + e(r^N - r^W))'(\bar{d} - B(p - er^W)) - c'(z + \bar{d} - B(p - er^W)) \geq p'(\bar{d} - B(p - er^W)) - c'(z + \bar{d} - B(p - er^W)) = \Pi^N(p, z, r^W)$. In other words, one can increase the price p by the difference in rebates $r^N - r^W$ and increase the profits. Finally, the second inequality follows from the feasibility of p^W and z^W .

7. Finally, we show the inequality for the government cost, i.e., $GC^N \geq GC^W$. We have: $GC^N = r^N e'E[\min\{d, q^N\}] = r^N \Gamma \geq r^W \Gamma = r^W e'E[\min\{d, q^N\}] = GC^W$, since we have shown in step 4 that $r^N \geq r^W$.

G. Proof of Proposition 7

After solving problem (11) for the cases with and without competition, we obtain $p^N - r^N = B^{-1}(\bar{d} - q_0^N) - \frac{e'(\Gamma - e'q_0^N)}{e'Be}e$ and $p^W - r^W = B^{-1}(\bar{d} - q_0^W) - \frac{e'(\Gamma - e'q_0^W)}{e'DXBe}XDe$. Therefore, we obtain:

$$p^N - r^N - (p^W - r^W) = B^{-1}(\bar{d} - q_0^N) - \frac{e'(\Gamma - e'q_0^N)}{e'Be}e - B^{-1}(\bar{d} - q_0^W) - \frac{e'(\Gamma - e'q_0^W)}{e'DXBe}XDe.$$

If the elasticity terms are symmetric (i.e., $B_{ij} = B_{lk}$ $i \neq j, l \neq k$ and $B_{ii} = B_{jj}$), we have $XDe = \mu e$ for some $\mu \geq 0$, and then $\frac{1}{e'DXB_e}XDe = \frac{\mu}{\mu e'B_e}e = \frac{1}{e'B_e}e$. Therefore:

$$\begin{aligned} p^N - r^N - (p^W - r^W) &= B^{-1}(q_0^W - q_0^N) - \frac{e'(q_0^W - q_0^N)}{e'B_e}e \\ &= B^{-1}(q_0^W - q_0^N) - \frac{e'BB^{-1}(q_0^W - q_0^N)}{e'B_e}e \\ &= (I - \frac{ee'B}{e'B_e})B^{-1}(q_0^W - q_0^N) \\ &= (I - \frac{ee'B}{e'B_e})(\frac{B^{-1}}{2} - X)(\bar{d} - Bc) \\ &= (I - \frac{1}{n}ee')(\frac{B^{-1}}{2} - X)(\bar{d} - Bc) \end{aligned}$$

Since all the elasticities are symmetric, we denote by $\alpha = B_{ii}$ and $\beta = B_{ij}$ $i \neq j$. Then, B^{-1} has *onlytwodifferentterms*: $\frac{\alpha+(n-2)\beta}{\alpha(\alpha+(n-2)\beta)-(n-1)\beta^2}$ and $\frac{-\beta}{\alpha(\alpha+(n-2)\beta)-(n-1)\beta^2}$ in its diagonal and off-diagonal respectively. Similarly, the expression for X is similar to B^{-1} but replacing α with 2α . Let $a = [\frac{B^{-1}}{2} - X]_{ii}$ and $b = [\frac{B^{-1}}{2} - X]_{ij}$ $i \neq j$, then $a \leq b$ since:

$$\begin{aligned} b - a &= \frac{\alpha + (n-2)\beta}{2(\alpha(\alpha + (n-2)\beta) - (n-1)\beta^2)} - \frac{2\alpha + (n-2)\beta}{2\alpha(2\alpha + (n-2)\beta) - (n-1)\beta^2} \\ &\quad - \frac{-\beta}{2(\alpha(\alpha + (n-2)\beta) - (n-1)\beta^2)} + \frac{-\beta}{2\alpha(2\alpha + (n-2)\beta) - (n-1)\beta^2} \\ &= \frac{-\beta}{2(\alpha - \beta)(2\alpha - \beta)} \geq 0. \end{aligned}$$

Note that $(I - \frac{1}{n}ee')(\frac{B^{-1}}{2} - X)$ is a symmetric matrix with $-\frac{n-1}{n}(b-a)$ and $\frac{1}{n}(b-a)$ in its diagonal off-diagonal respectively. Note also that each row sums to zero. We have:

$$\begin{aligned} p^N - r^N - (p^W - r^W) &= (I - \frac{1}{n}ee')(\frac{B^{-1}}{2} - X)(\bar{d} - Bc) \\ &= -(b-a)((\bar{d} - Bc) - e\frac{e'(\bar{d} - Bc)}{n}) \\ &= -(b-a)((\bar{d} - (\alpha - \beta)c - \beta e'ce) - e\frac{e'(\bar{d} - (\alpha - \beta)c - \beta e'ce)}{n}) \\ &= -(b-a)((\bar{d} - (\alpha - \beta)c) - e\frac{e'(\bar{d} - (\alpha - \beta)c)}{n}). \end{aligned}$$

Observe that the average (or sum) of all the changes in effective price is zero. If \bar{d} and c are symmetric, we have $p^N - r^N - (p^W - r^W) = 0$ so that $q^N = q^W$. As a result, this leads to the same consumer surplus and social welfare. If there are asymmetries in \bar{d} or c , one can see that $p_i^N - r^N - (p_i^W - r^W)$ is positive in the product with the lowest $\bar{d}_i - (\alpha - \beta)c_i$, and negative for the segment with the highest one. So, if there are asymmetries only in \bar{d} , the products with lowest \bar{d}_i will have $p_i^N - r^N > p_i^W - r^W$, and the opposite for the one with highest \bar{d}_i . A similar result applies for asymmetries in c , but with the signs reversed. Finally, note that we have:

$$e'(p^N - r^N - (p^W - r^W)) = -(b-a)(e'(\bar{d} - (\alpha - \beta)c) - e'e\frac{e'(\bar{d} - (\alpha - \beta)c)}{n}) = 0.$$

H. Proof of Corollary 1

We show the inequality for the expected consumer surplus in the symmetric case, i.e., $CS^N \leq CS^W$.

One can write the expected consumer surplus as a function z , as follows:

$$\begin{aligned} CS(z) &= \frac{1}{2} \left(\bar{d} - B(p - er) \right)' B^{-1} \left(\bar{d} - B(p - er) - \Theta(z) \right) + \frac{1}{2} e' D_{B_{ii}^{-1}} \Xi(z) \\ &= \frac{1}{2} \left(\bar{d} - B(p - er) - \Theta(z) \right)' B^{-1} \left(\bar{d} - B(p - er) - \Theta(z) \right) + \frac{1}{2} e' D_{B_{ii}^{-1}} \Xi(z) \\ &\quad + \frac{1}{2} \Theta(z)' B^{-1} \left(\bar{d} - B(p - er) - \Theta(z) \right) \\ &= \frac{\Gamma^2 e' B^{-1} e}{2n^2} + \frac{1}{2} \left(e' D_{B_{ii}^{-1}} \Xi(z) + \frac{\Gamma}{n} e' B^{-1} \Theta(z) \right). \end{aligned}$$

The last equality follows from the symmetry of the suppliers so that each firm sells Γ/n , in expectation. Therefore: $E[\min\{d, q\}] = \bar{d} - B(p - er) - \Theta(z) = \frac{\Gamma}{n} e$. We define the following function of the vector z : $\Omega(z) = \sum_{i=1}^n e_i \int_{z_i}^{A_i} \epsilon_i f(\epsilon_i) d\epsilon_i$. We next compute the gradient of the expected consumer surplus with respect to z :

$$\begin{aligned} \nabla_z CS &= \frac{1}{2} D_{B_{ii}^{-1}} (\Omega(z) + D_{\bar{F}(z)} z) - \frac{1}{2} \frac{\Gamma}{n} B^{-1} \bar{F}(z) \leq \frac{1}{2} D_{B_{ii}^{-1}} (\Omega(z) + D_{\bar{F}(z)} z) - \frac{1}{2} \frac{\Gamma}{n} e' D_{B_{ii}^{-1}} D_{\bar{F}(z)} \\ &= \frac{n}{2} B_{11}^{-1} e \int_{z_1}^{A_1} \left(\epsilon - \frac{\Gamma}{n} \right) f_1(\epsilon) d\epsilon \leq \frac{n}{2} B_{11}^{-1} e \int_{z_1}^{A_1} \left(A_1 - \frac{\Gamma}{n} \right) f_1(\epsilon) d\epsilon \leq 0_{n \times 1}. \end{aligned}$$

The first inequality follows from the fact that $B^{-1} \geq 0$ and the second equality is from the symmetry of the problem. The second inequality uses the fact that ϵ is bounded by A_1 and the third inequality follows from the assumption $e'A \leq \Gamma$. Finally, since the optimal prices satisfy $\bar{F}_i(z_i) p_i = c$, z is a non-decreasing function of p . Therefore, $p^N \geq p^W$ implies that $z^N \geq z^W$ and this concludes the proof.

I. Proof of Proposition 8

We start by presenting the proof for the case of asymmetric suppliers.

- $q^N \leq_1 q^W$

The total production quantities are given by:

$$e'q^N = \frac{k'Be + 2e'q_0^N}{4} \leq \frac{k'DXBe + 2e'DXq_0^N}{2} = e'q^W.$$

Therefore, it follows that at least one component is smaller, i.e., $q^N \leq_1 q^W$.

- $p^N - r^N \geq_1 p^W - r^W$

We have shown that $e'q^N \leq e'q^W$ and therefore $e'B(p^N - er^N) \geq e'B(p^W - er^W)$. Since $e'B > 0$, then $p^N - r^N \geq_1 p^W - r^W$.

- $CS^N \leq CS^W$

For ease of the demonstration, we consider the case where B is symmetric so that $B_{ii} = B_{jj}$ and $B_{ij} = B_{kl}$ for all $i \neq j, k \neq l$ (products with the same self and cross elasticities).

$$\begin{aligned}
CS^N &= \left(q_0^N + \frac{Be k' Be - 2e' q_0^N}{2} \frac{B^{-1}}{2e' Be} \right)' \frac{B^{-1}}{2} \left(q_0^N + \frac{Be k' Be - 2e' q_0^N}{2} \frac{B^{-1}}{2e' Be} \right) \\
&= \left(q_0^N + \frac{Be k' DX Be - 2e' DX q_0^N}{2} \frac{B^{-1}}{2e' DX Be} \right)' \frac{B^{-1}}{2} \left(q_0^N + \frac{Be k' DX Be - 2e' DX q_0^N}{2} \frac{B^{-1}}{2e' DX Be} \right) \\
&\leq \left(2DX q_0^N + DX Be \frac{k' DX Be - 2e' DX q_0^N}{2e' DX Be} \right)' \frac{B^{-1}}{2} \left(2DX q_0^N + DX Be \frac{k' DX Be - 2e' DX q_0^N}{2e' DX Be} \right) \\
&= CS^W
\end{aligned}$$

- $r^N = r^W$

We have:

$$r^N = \frac{k' Be - 2e' q_0^N}{2e' Be} = \frac{k' B \beta e - 2\beta e' q_0^N}{2e' B \beta e} = \frac{k' BX De - 2e' DX q_0^N}{2e' BX De} = r^W.$$

The second equality follows from multiplying the numerator and the denominator by any scalar $\beta \neq 0$. The third equality is obtained by using the fact that the suppliers are symmetric: $XDe = \beta e$ for some $\beta > 0$.

- $p^N \geq p^W$

We have:

$$\begin{aligned}
p^N &= B^{-1}(\bar{d} - q_0^N) + \frac{k' Be - 2e' q_0^N}{4e' Be} e = B^{-1}(\bar{d} - q_0^N) + \frac{k' Be - 2e' q_0^N}{2e' Be} \frac{e}{2} \\
&= B^{-1}(\bar{d} - q_0^N) + \frac{k' DX Be - 2e' DX q_0^N}{2e' DX Be} \frac{e}{2} \quad \left(r^N = r^W \right) \\
&\geq B^{-1}(\bar{d} - 2DX q_0^N) + \frac{k' DX Be - 2e' DX q_0^N}{2e' DX Be} \frac{e}{2} \quad \left(DX \geq \frac{I}{2} \right) \\
&\geq B^{-1}(\bar{d} - 2DX q_0^N) + \frac{k' DX Be - 2e' DX q_0^N}{2e' DX Be} XBe \quad \left(XB \leq \frac{I}{2} \right) \\
&= p^W
\end{aligned}$$

- $p^N - r^N \geq p^W - r^W$ and $q^N \leq q^W$ follow directly from the asymmetric case shown above.
- $\Pi^N \geq \Pi^W$

The profit in the monopolistic setting is given by:

$$\begin{aligned}
\Pi^N &= \left(q_0^N + \frac{Be k' Be - 2e' q_0^N}{2} \frac{B^{-1}}{2e' Be} \right)' \left(B^{-1} q_0^N + \frac{e k' Be - 2e' q_0^N}{2} \frac{B^{-1}}{2e' Be} \right) \\
&= q_0^N B^{-1} q_0^N + e' q_0^N \frac{k' Be - 2e' q_0^N}{2e' Be} + \frac{(k' Be - 2e' q_0^N)^2}{16e' Be}.
\end{aligned}$$

The profit in the competitive environment is given by:

$$\begin{aligned}
\Pi^W &= \left(2DXq_0^N + DXBe \frac{k'DXBe - 2e'DXq_0^N}{2e'DXBe} \right)' \left(2Xq_0^N + XBe \frac{k'DXBe - 2e'DXq_0^N}{2e'DXBe} \right) \\
&= \left(2DXq_0^N + DXBe \frac{k'Be - 2e'q_0^N}{2e'Be} \right)' \left(2Xq_0^N + XBe \frac{k'Be - 2e'q_0^N}{2e'Be} \right) \quad (\text{use of symmetry}) \\
&= 4q_0^{N'} XDXq_0^N + 4e' BXDXq_0^N \frac{k'Be - 2e'q_0^N}{2e'Be} + \frac{(k'Be - 2e'q_0^N)^2}{16(e'Be)^2} 4e' BXDXBe.
\end{aligned}$$

We next show that $B^{-1} \geq 4XDX$. Since $D - B$ and B^{-1} are non-negative matrices, we obtain:

$$\begin{aligned}
(D - B)'B^{-1}(D - B) \geq 0 &\Leftrightarrow (D + B)'B^{-1}(D + B) \geq 4D \\
&\Rightarrow B^{-1}(D + B) \geq 4XD \quad (X = (D + B)^{-1} \geq 0) \\
&\Rightarrow B^{-1} \geq 4XDX \quad (X \geq 0)
\end{aligned}$$

Therefore, $\Pi^N \geq \Pi^W$.

- $GC^N \leq GC^W$

Since $r^N = r^W$ and $q^N \leq q^W$, we have: $GC^N = q^N r^N \leq GC^W = q^W r^W$.