CEEPR Center for Energy and Environmental Policy Research

Costs of Aggregate Hydrocarbon Reserve Additions

by
M.A. Adelman and G.C. Watkins

02-011 WP December 2002

A Joint Center of the Department of Economics, Laboratory for Energy and the Environment, and Sloan School of Management

COSTS OF AGGREGATE HYDROCARBON RESERVE ADDITIONS[1]

M. A. Adelman,
Massachusetts Institute of Technology
and
G.C. Watkins.
King Fahd University of Petroleum and Minerals
and University of Aberdeen

Introduction. Typically, investment in petroleum exploration and development yields a joint product: reserves of oil and natural gas. The reserve additions are separable, but the expenditures generating reserve additions cannot be neatly partitioned between the two hydrocarbon sources.

It is not surprising that some way of aggregating the results of exploration and development activity has been sought to relate total effort (cost) to results (reserve additions). The most popular technique has been to convert natural gas to 'oil equivalent' at a fixed ratio based on physical thermal content or on some thermal value content implied by relative wellhead prices at a given point in time.

There are two major problems with fixed coefficients. First, thermal conversion assumes oil and gas are perfect 'thermal' substitutes in the marketplace – on the side of demand and also supply. But clearly they are not. Second, relative values of oil and gas change over time.

In what follows, we highlight problems created by aggregation using fixed conversion coefficients (Section 1). We then offer an economic index approach as an alternative, one that recognizes changing relative values of oil and gas over time (Section 2). This aggregation technique - the Divisia index - is applied to US reserve and in situ price data from 1982 to year 2001 to derive implicit shifts in unit costs of aggregated oil and gas reserve additions; these results are compared with those from the traditional fixed coefficient measures (Section 3). Concluding remarks are in Section 4.

To anticipate: we find that estimates of aggregated reserve quantities and of unit costs of aggregate reserve additions are materially affected by the aggregation technique employed. We argue that the Divisia approach is superior to the usual fixed coefficient methods by allowing for time-varying imperfect substitutability between oil and gas, rather than assuming perfect substitutability.

[1] We thank Ernst Berndt (MIT) and James Smith (Southern Methodist University) for helpful comments – the usual disclaimer holds. We are greatly indebted to Jie Yang (MIT) for skillful and devoted assistance. An earlier version of this paper was given at the 25th IAEE International Conference, Aberdeen, June 2002 (Adelman and Watkins [2002a]).
.

We first show that adding together oil and gas at a fixed relation makes little sense in economic theory, since oil and gas are only partially interchangeable in respect of either demand or supply. They do compete in some markets, not in others. Any comparison of values per British thermal unit (BTU) is strongly affected by different energy-using apparatus and delivery.

Our alternative approach to aggregation requires input of the annual average value of oil reserves and of natural gas reserves, separately valued by market forces in situ, as they are at the wellhead. The time series of reserve prices we employ is an updated and revised version of our earlier results, based on regression analysis of market transactions, the sale of oil and gas reserves in the USA.

There is public information on the US national total of gross reserves added in each year. Multiplying these totals by their in situ value yields the estimated average market value of the physical reserve additions. The industry that makes the finding-development investment is competitive. Therefore the values of oil and gas reserves in situ provide a measure of the marginal costs of creating additional oil and gas reserves. Average costs of oil and of gas do not exist, but marginal costs do, and are calculated here.

Our next task, the computation of a combined quantity of oil-plus-gas, is necessary, because most hydrocarbon deposits contain both. Nature presents many mixed bags to be found and developed. Investment moves toward more profitable deposits, and toward that hydrocarbon which is more valuable to develop. In short, liquids and gas are joint products, in variable proportions, of finding-development investment. The proportions are changeable over time, and indeed we wish to capture those changes; hence it makes little sense to calculate a once-and-for-all ratio of gas to oil.

Combining quantities of oil and gas in an economically meaningful way is an index number problem. The proper weights for aggregating the changes in the two reserve quantities over time are a function of changes in the respective estimated expenditures on reserve additions.

1. Perspectives on 'Oil Equivalent'

"Oil equivalent" is widely used to measure the total hydrocarbons contained in a reservoir, or produced by a company, or within some area, and so forth. Natural gas is used as a proxy for liquids, or vice versa, converting at 5,500 cubic feet per barrel or some other fixed coefficient.[2] The rationale was that 1000 cubic feet of gas measured at 1000 BTUs per cubic foot is 1 million BTUs; a barrel of crude oil contains about 5.5 million BTUs. Reports in the press, by consulting firms, etc, have used 6:1. Companies have generally stayed close to the 5.5:1 thermal equivalence. Total has used 5.487 mcf

[2] DOE *Monthly Energy Review* Appendix Tables A4, A6.

per barrel, Chevron-Texaco and Shell Canada, 6.0 per barrel. But Suncor Energy, Pan Canadian (now Encana), and Scotia Capital have used 10 mcf per barrel, explicitly called economic equivalence. The market-based foundation for that relation seems to have been a ratio of upstream prices per BTU of oil compared with gas at one point in time, a time when gas prices per BTU were close to half those for oil.

We have deplored the use of “oil equivalent” (for example, Adelman and Watkins [1996, p1, p34]). The late John Lohrenz, to whom all interested in oil economics are much indebted, wrote a brief paper: "In Situ Gas-to-Oil Equivalence 6 MCF/barrel? Aw C'mon!”

“Oil equivalent” is not an inexact or imperfect measure. Because it lacks economic content, it is simply no measure of economic aggregates. Lord Browne of BP-Amoco has said of oil equivalent: “No such thing exists.” (*Petroleum Intelligence Weekly*, March 23, 1998, p.7). In econometric work, explaining differences among companies or over time by differences or changes in “oil equivalents” means writing errors into the independent variables, a pollution hard to remove.

The Scotia Group Inc., Dallas, has compiled the sales for over twenty years of properties producing oil and gas. (We are indebted to them for the data but are entirely responsible for its use.) In 2001 there were 55 usable observations, totaling some $7 billion paid, for 349 million bbl of liquids and 3343 million mcf of gas. Converting at the thermal equivalence of 5.5:1 gives 957 million "boe," so the unit value is seemingly $7.31/boe.

We can get a measure of each hydrocarbon’s reserve value by regressing transaction values on volumes of purchased reserves. For the Scotia Group data for 2001, the estimated value of in situ oil is $4.38 per barrel, of gas $1.67 per mcf; both estimates are statistically significant[3]. The gas:oil ratio from the regression equation is only 2.6:1, not 5.5:1. The market prices of oil and gas at the wellhead for 2001 were $21.84 per barrel and $4.12 per mcf, respectively, a ratio of 5.3, one that happens to be close to thermal equivalent (*Monthly Energy Review*).

For many years, petroleum engineers have adhered to a working rule in North America, that the in-ground value of a developed barrel or mcf is about one-third of the wellhead price[4]. In situ gas in year 2001 was worth 0.4 of the wellhead price, a barrel of oil only 0.2. Once we have escaped the irrational tyranny of “oil equivalent”, we are able to ask why.

The working rule of 3:1 really consists of two parts. First, about one third of the gross wellhead value covers operating costs, royalties, and taxes[5]. For the owner, considering whether to produce the reserve or to sell it, the comparison of benefits is of

[3]For details on estimation of these values, and for earlier years, see Adelman and Watkins [2002b].
[4]We also found previously that the rule was not a bad fit; see Adelman and Watkins [1996, p34].
[5] Adelman and Watkins [1996, p4].

2:1. Denoting by P the *net* wellhead market value of a barrel of oil or mcf of gas, and by V the sales value of a reserve, we have shown elsewhere (Adelman and Watkins [1995, p669]) that the relation is approximated by:

$$V = Pa/(a+i-g) \quad (1)$$

where a = the production decline rate of the reserve being sold, i = the rate of discount on future receipts, and g = the expected rate of increase or decrease of the market price. All three components of the denominator are expressed as annual rates of change. If the (net) price is expected to increase at the rate of discount, as is still sometimes argued, obviously V = P, the value of a unit in-ground is equal to a net value at the wellhead. But if we take the decline rate, a, as set by local geology and technique, i as an economy-wide fact, and make no presumption about g, then we can solve for it:

$$g = i + a(1-(P/V)) \quad (2).$$

For North American oil, the average current annual decline rate approximates nine per cent, for natural gas 11 per cent, based on the ratio of remaining reserves to production. If the discount rate is 15 per cent, then the reserve values (V) of $4.38 per barrel for oil and $1.67 per mcf for gas suggest that buyers and sellers expected gas prices to increase annually by about eight per cent. If the discount rate were 10 per cent, the expected increase in gas prices would be three per cent. For oil, with a discount rate of 15 per cent oil prices would be expected to decline annually at around six per cent; if the discount rate were 10 per cent, the decline would be 11 per cent. These estimates use the US (net) wellhead prices (P) cited above[6].

Of course these are two rough guesses. But the reserve market was betting that that gas prices would show an upward trend, while oil prices would drop. Differing expectations were not unreasonable for rational buyers and sellers to hold, given (a) a stronger gas market, and (b) in world oil, the insecurity of the cartel in maintaining a price so far above its investment cost. These are two different markets, with two price-determining systems and degree of price risk, even though oil and natural gas do compete in some end uses[7].

Better data would yield better numbers for oil and gas reserve values. But values in any year are inherently uncertain and affected by transitory factors. There is more interest in looking for persistent changes over time in oil and gas in-ground values. For example: if it were really becoming more difficult and expensive to find oil and gas in the ground, this should be reflected in rising in situ values per barrel or mcf; and vice versa. We comment further on this in Section 3.

---

[6]After deducting an allowance for extraction cost of one third of the price. The results are sensitive to the decline rates. For example, significantly higher decline rates for natural gas would result in expectations of declining gas prices.
[7]This issue is pursued further with data for several years in Adelman and Watkins [2002b].

Our main concern here, however, is that these oil and gas in situ values bear little relation to the conventional fixed conversion factors cited earlier for purposes of aggregating oil and gas reserves. The fixed coefficient technique is flawed because it is not grounded in market values[8]. Below, we offer an alternative approach to aggregation based on economic indices.

2. An Economic Index Approach.

Energy is only used in conjunction with some form of energy using equipment. Hence, the demand for energy is derived from the demand for the services provided by energy using equipment, not for itself. Consumers generally do not buy BTUs, producers do not sell BTUs[9]. Yet it is commonplace for energy aggregates to be expressed as BTUs. This exercise can be useful as an accounting yardstick, but has little economic meaning. It ignores the fact that energy sources differ in quality. Even with universal competitive markets, energy prices per BTU would vary among energy types by reflecting different attributes such as weight, cleanliness, end use, conversion costs, safety, ease of storage and the like.

The sequence and content of our discussion below in large measure follows that in Berndt [1978, pp. 238-48] but is confined to crude oil and natural gas. If only heat content mattered, and oil and gas were perfectly substitutable, oil and gas prices for the end user indeed would be much the same per BTU. But this is far from the case. Differences in end user BTU prices of oil and gas suggest consumers value oil and gas on more than just heat content.

Thus, in 2001 oil delivered for power generation in the US cost $4.45 per million BTU, and gas cost $3.92 per million. But oil exiting a refinery as (a) motor gasoline cost $7.09 cents per million BTU; (b) aviation gas, $10.47; (c) home heating oil, $5.45; (d) Diesel fuel, $5.65 cents.[10] In other words, much or most of the value in a barrel of oil lay in the light products into which it was refined. These values change up and down, and one year's relations may be very different from the year before and the year after. This market complexity is inescapable. Oil and gas equivalence at any fixed rate simply adds confusion.

Downstream differences reverberate upstream and can be amplified or compressed by differences in transportation costs per BTU. Natural gas is more expensive to transport on a BTU basis than oil. Hence if downstream oil prices (expressed in terms of BTUs) exceeded natural gas, the difference back in the field would

---

[8] Berry, Hasan and O'Bryan[1998] looked at the relationship between company equity values and whether reserves were aggregated using a thermal or value conversion factor. They found in favour of thermal conversion on the grounds of reduced standard errors, but this finding was predicated on wellhead rather than in situ prices.

[9] Although most natural gas purchase prices are calibrated with BTU content, reflecting a predominance of sales for space heating and boiler use .

[10] Monthly Energy Review, Tables 9.4, 9.10, and A2.

be magnified, and vice versa if downstream natural gas prices exceeded oil.

Variations in oil and gas BTU prices across time and across locations demonstrate that oil and gas are certainly far from perfect thermal substitutes in the market place. It follows that any aggregation of oil and gas quantities using BTU conversion factors fails to capture market valuation by end users: BTU based aggregates lack economic content.

In the context of oil and gas supply, the economic index number approach to this problem would be to weight quantities of oil and gas in a way that would reflect their respective relative marginal cost (supply price) per unit of measurement. In competitive markets, prices would equal marginal costs and respective upstream prices would measure the relative unit worth of oil and gas[11].

Certainly, petroleum markets (as with most markets) fall short of the perfectly competitive standard. Nevertheless, deregulation in North America has been pervasive and crude oil and natural gas prices during the last two decades or so provide reasonable approximations to suitable index weights. Moreover, if relative prices were used, any consistent biases in the respective oil and gas valuations would be eliminated.

How might oil and gas prices be incorporated in index number formulae that seek to aggregate quantities? Call the quantity of oil and natural gas at time t $Q_{o,t}$ barrels and $Q_{g,t}$ mcf respectively, with corresponding prices $P_{o,t}$ per barrel and $P_{g,t}$ per mcf; each price can be deflated by a suitable price index to express prices in real terms[12]. Total expenditure on oil or gas, EXP_t, is the product of the respective prices and quantities:

$$EXP_t = Q_{o,t}P_{o,t} + Q_{g,t}P_{g,t} \qquad (3).$$

If the price of oil were adopted as a numeraire, one way of constructing an oil and gas quantity index, QE_t, would be to weight the respective quantities by the relative prices:

$$QE_t = Q_{ot} + (P_{g,t}/P_{o,t})\, Q_{gt} \qquad (4).$$

According to (4), one unit of oil is equivalent to $(P_{g,t}/P_{o,t})$ units of gas[13]. If the BTU prices of oil and gas were the same and oil and gas quantities were expressed in BTUs, then the relative price would be unity and expression (4), the aggregate quantity index, would indeed be the simple BTU sum of the two commodities. *An economic aggregate of oil and gas supply would only be identical with a BTU aggregate if BTU prices (or costs) were the same and oil and gas were perfectly substitutable.* Such parity pricing and substitutability are a special case that seldom if ever holds. But to the extent that the relative prices of oil and gas do measure consumer preferences, expression (4) is

[11] With imperfect competition, the weights would best reflect the marginal physical revenue product.
[12] It is immaterial at this stage whether these prices are wellhead or insitu prices.
[13] Note that if the same deflator applied to both oil and gas prices (as would be normal), then the price ratio would be invariant as to whether prices were expressed in nominal or real terms.

clearly preferable to BTU summation.

Note that to preserve the total expenditure relation given by (3), the corresponding aggregate oil and gas price index, PE_t, given quantities defined by (4), can be computed as total expenditure divided by the aggregate oil and gas quantity given by (4):

$$PE_t = EXP_t/QE_t \qquad (5).$$

However, expression (4) is itself restrictive, treating one unit of oil as perfectly substitutable with ($P_{g,t}/P_{o,t}$) units of gas. Such strict proportionality of substitution is unlikely. A preferred indexing technique would be a more general one. Berndt mentions the Cobb-Douglas index (Berndt [1978, p245], but points out that it assumes that substitution possibilities and expenditure shares are constant. These restrictions are not attractive. More promising are general index number formulae based on the classic work of Fisher [1922], notably developed by Diewert, who especially emphasized the discrete approximation to the continuous Divisia index (Diewert [1976]).

The comparison of one year with another is an old problem. A frequently used device is the Laspeyres index, where the weights of the base year are multiplied by the prices of each later year. In contrast the Paasche index uses the weights of the end year. The infirmities of both indexes have long been known: taking the relations among products at one moment, and generalizing them backward or forward through time. Hence the appeal of a hybrid measure such as Fisher's "Ideal" index, which in effect is the geometric mean of Laspeyres weights and Paasche weights.

In more detail: the most noteworthy properties that aggregate indices should satisfy are that they be single valued, separable and homothetic. Single valued means that for a given set of prices and quantities there is a unique solution for the aggregate quantity index. Separability is the requirement that buyer or seller preferences among the aggregated quantities should be independent of the other quantities that lie outside the aggregated set. Homotheticity requires that when all components of the aggregate increase by a constant factor, the aggregate index also increases (at least within a small range of component changes)[14].

Divisia indices[15] incorporate these desirable properties. Theoretically, Divisia indices are defined in terms of differential equations in continuous time, where the rate of change in the aggregate quantity index is the (instant) expenditure weighted sum of the rate of change of each component. Practical applications require approximation. Laspeyres, Paasche and (Fisher) Ideal indices can all approximate true Divisia indices over small price changes. The Tornquist or Fisher discrete approximation we use is a good discrete approximation over larger price changes. The weights used for discrete

[14]A special case of this would be constant returns to scale: the aggregate index would vary by the same (constant) as the components.
[15] Named after the French economist Francois Divisia who introduced the technique in 1921.

time intervals are arithmetic averages of expenditure shares in two adjacent periods, and the continuous rate of change in quantities is approximated by differences between one year and the next. Diewart [1976] has shown that the discrete Divisia index permits varying substitution possibilities without imposing parameter restrictions.

Divisia indices are chain linked, not binary. A binary index of values between two time periods depends only on information at the beginning and end of a period of time. Chain linked indices depend on the path taken by prices and quantities during the interval.

In our case of two commodities, oil and gas, a Divisia index for an aggregated quantity, QE_t at time t, would be derived from the expression:

$$\ln QE_t - \ln QE_{t-1} = a_{o,t}(\ln Q_{o,t} - \ln Q_{o,t-1}) + a_{g,t}(\ln Q_{g,t} - \ln Q_{g,t-1}) \qquad (6)$$

where the a's are average expenditure shares for adjacent years, namely:

$$a_{o,t} = 1/2\ (w_{o,t} + w_{o,t-1}) \text{ where } w_{o,t} = Q_{o,t}P_{o,t} / EXP_t \qquad (7).$$

The expressions for $a_{g,t}$ and $w_{g,t}$ are analogous (in the two commodity case, of course $w_{g,t}$ is 1 - $w_{o,t}$ and a_{gt} is 1 - a_{ot}). The expenditure shares typically will vary over time; supply prices enter the index via the expenditure share weights. Note that the antilog of equation (6) is the ratio of the aggregate quantities in successive periods. Calculations of changes in Divisia quantities are invariant to whether prices are expressed in real or nominal terms, as long as the same deflator applies to each price component.

The discrete Divisia index treats the percentage change in the aggregate oil and gas quantity index as the weighted average of the percentage change in the individual quantities of oil and gas, where the weights are two period moving average expenditures or cost shares. If the cost shares were constant – which is seldom - the discrete Divisia index would collapse to a Cobb-Douglas index.

When quantities of both oil and gas increase between adjacent periods, the Divisia quantity given by (6) will also increase (the homotheticity property). Suppose the percentage growth in the quantity of oil were matched by the same percentage decline in the gas quantity. As long as the weighted average expenditure share for oil exceeded that for gas, the Divisia quantity would rise. But whether that inequality would hold depends on what happens to prices. A decline in gas quantities would tend, other things equal, to reduce gas expenditure shares. But it is conceivable that an increase in the price of gas could be such as to reduce Divisia quantities, even though the quantity of oil may have increased. Hence a change in Divisia quantities is dependent not only on underlying changes in the quantities of its components, but also on what has happened to relative prices. Decomposing the sources of change in a Divisia index is not a straightforward exercise.

The Divisia index has been used extensively in energy demand analysis, aggregating over different energy types. However, Berndt has rightly cautioned that aggregate energy indexing may be difficult to interpret and not well defined, although the economic theory of aggregation does provide a rigorous framework (Berndt, op cit., p248).

In Section 3, we shall show how quantity and price (cost) information over a period of time may be employed using the Divisia technique to provide a quantity index of reserve additions that can be compared with the change in expenditures to see whether aggregate costs are rising or falling, and to what extent. We can also create an aggregated quantity to derive an absolute measure of unit cost for purposes of comparison with unit cost estimated via the oil equivalent approach discussed earlier.

Below we provide an illustration of the calculations employed, using assumed data for two periods.

An Illustration. Assume the following values:

$Q_{o,t}$ = 500 mmbbls, $Q_{g,t}$ = 500 bcf, $P_{o,t}$ = \$3.00 bbl, $P_{g,t}$ = \$1.00 mcf
$Q_{o,t-1}$ = 400 mmbbls, $Q_{g,t-1}$ = 600 bcf, $P_{o,t-1}$ = \$2.75 bbl, $P_{g,t-1}$ = \$0.80 mcf.

From (3), expenditures on reserve additions, EXP_t = 500 x \$3 + 500 x \$1 = \$2 billion, and from (7) $w_{o,t}$ = 1500/2000 = 0.75 and $w_{g,t}$ = 0.25. For the previous period we have EXP_{t-1} = 400 x \$2.75 + 600 x \$0.80 = \$1.580 billion, while $w_{o,t-1}$ = 1100/1580 = 0.70 and $w_{g,t-1}$ = 0.30.

Hence from (7) $aw_{o,t}$ = 0.5(1.45) = 0.725 and $aw_{g,t}$ = 0.5 (0.55) = 0.275.

ln $Q_{o,t}$ = 6.215 and ln $Q_{o,t-1}$ = 5.991, yielding a difference of 0.224;
ln $Q_{g,t}$ = 6.215 and ln $Q_{g,t-1}$ = 6.397, yielding a difference of -0.182.

From (6) the Divisia index at time t is:

DIV_t = 0.725(0.225) + 0.275 (-0.182) = 0.1130.

Taking the antilog of the Divisia index yields a growth rate in aggregate reserves of 11.9 per cent. Expenditures rose by a factor of 2/1.580 = 26.6 per cent. Hence aggregate reserve additions became more expensive in year t compared with the previous year, with unit costs rising by a factor of 1.266/1.119 = 13 per cent.

In comparison, barrels of oil equivalent (boe) calculations using a BTU conversion factor 5.5 mcf per barrel give a boe in year t of 591 barrels, 509 barrels in year t-1. The percentage increase is 16.1 per cent. The percentage increase in expenditures remains at 26.6 per cent. Hence the increase in aggregate unit costs of reserve additions using the BTU conversion would be 9 per cent. (1.266/1.161), compared with the Divisia number of 13 per cent. The BTU conversion technique in this instance markedly underestimates the increase in aggregate economic costs. A different set of data could result in the BTU technique overestimating aggregate economic costs.

Aggregate Divisia Quantities. The Divisia calculations are couched in terms of indexes, measuring changes over time, not absolute quantities. We can create an absolute Divisia quantity as an adjunct to an index value by applying the Divisia index to a specified benchmark volume. We call such quantities 'Divisia barrels of oil equivalent' (Dboe). They can be used to compare estimates of absolute unit costs from the Divisia approach with those from using fixed coefficients, as opposed to only comparing growth rates. The comparison assumes a common point of departure.

In our illustration, suppose we adopt as a benchmark the 509 barrels of boe in year t-1. Then Divisia barrels in year t would be 509 x 1.12 = 570 Dboe.

The unit cost of reserve additions in year t-1, using the benchmark quantity, would be expenditures of $1580 million divided by 509 million boe, yielding $3.10 per boe. The unit cost of reserve additions in year t using the Divisia procedure would be $2000/570 or $3.51 per Dboe, an increase of 13 per cent (as also calculated earlier using the growth rates). In contrast the unit cost using the 5.5 thermal conversion factor is $3.38 per boe in year t. While any Dboe quantity is a physical fiction, it is not an economic fiction.

We now turn to applying these ideas to data on US oil and gas reserve additions for the period 1982 to 2001, in conjunction with corresponding estimates of in situ prices.

3. Application to US Data, 1982-2001

Our concern is to get a handle on meaningful aggregation of oil and gas reserve additions and their aggregate costs. To create an oil and gas quantity index of reserve additions we use the Divisia approximation given by expression (6). This expression yields the change in the index between two adjacent periods. The calculation can be repeated for other adjacent periods, providing a chained index.

Data on reserve addition volumes measured in barrels of liquids and cubic feet of natural gas are normally unambiguous, although consistency of reserve definition is a perennial issue. Development investments related to reserve additions are more problematical. As mentioned earlier, oil and gas reserves are normally joint products. But sometimes natural gas found with oil is not marketable. Here oil related costs would become less ambiguous, by default. Conversely, the yield of light oil (condensate) found with natural gas could be so lean as to make recovery uneconomic, enabling gas costs to be identified. However, this is not the norm. Moreover, finding costs are not only joint between oil and gas but may also be joint across discoveries.

This is where information on oil and gas reserve prices – in situ prices - is valuable. A price of reserves, like the price of a widely traded stock, or of NYMEX spot crude oil prices, would measure the market price and thus the marginal cost or supply price of reserves. Hence the estimates of in situ prices of oil and gas from an earlier work, now extended [16].

To contrast and compare aggregation techniques, we create three sets of *aggregated* annual oil and natural gas reserve additions, using official US data for the period 1982-2001. Two sets are calculated by converting natural gas to 'oil equivalent' employing fixed conversion factors of 5.5 mcf per barrel and 10 mcf per barrel respectively. Recall that the 5.5 coefficient represents conversion on a thermal basis, and is hereafter referred to as 'thermal conversion'. The coefficient of 10 is one that has been used by some in industry (see earlier) to represent an oil price per BTU close to double that for gas implicit in thermal value. It is analogous to expression (4) above, although the ratio is fixed, not time varying. Hereafter it is called 'price ratio conversion'. The third set uses the Divisia index method (see equation (6)).

The procedure we follow below is to first set out the basic data employed. Next we compare changes in quantities derived from the three aggregation techniques. We then relate these changes to changes in total estimated expenditures on oil and gas reserve additions. This enables us to estimate and compare changes in unit cost relating to the three sets of aggregated reserve additions. Finally, we attempt to compare absolute unit costs, as opposed to changes in unit costs, to provide another perspective on differences among the three techniques.

The Data. The basic data used in making these calculations are shown in Table 1. For aggregation of quantities using fixed coefficients, only reserve additions (columns (2) and (4) are relevant. Calculation of rates of growth in Divisia quantities requires in situ price data (columns (3) and (5)) as well as reserve additions. The prices are drawn from estimates provided in Adelman and Watkins [2002b][17], and are nominal (not inflation adjusted) prices.[18] Although we refer to these as prices, recall that in the context of this analysis we are using them to represent marginal costs of reserve additions (supply prices).

In passing we observe there is no underlying pattern in the in situ prices over the 20 year period 1982-2001 which suggests growing scarcity, or abundance. This issue is discussed further in Adelman and Watkins [2002b].

Estimated expenditures on reserve additions are the sum of expenditures on oil and natural gas reserve additions (col (6), Table 1). Oil reserve expenditures are the product of oil reserve additions and the estimated in situ price of oil reserves; gas reserve expenditures are the product of gas reserve additions and the in situ price of gas reserves. Total expenditures are invariant to aggregation technique.

[16] See Adelman and Watkins [1996] and [2002b].

[17] Adelman and Watkins [2002b] include the prices from the earlier paper (Adelman and Watkins [1996]) some revised.

[18] Thus our estimated expenditures and later estimates of unit costs are nominal, not price adjusted. Recall that Divisia quantity changes are invariant to whether expenditures are in real or nominal terms, as long as the same deflator applies to each component – here oil and natural gas.

Table 1
Basic Data

1	2	3	4	5	6
Year	*Volumes of Oil Reserve Additions (mmbbls)*	*Insitu Price of Oil ($/bbl)*	*Volumes of Gas Reserve Additions (bcf)*	*Insitu Price of Natural Gas ($/mcf)*	*Total Expenditures ($million)*
1982	2256	5.58	17288	0.95	29012
1983	4302	4.31	14253	0.66	27949
1984	4266	6.95	14409	0.86	42092
1985	4076	4.90	11891	0.84	29988
1986	2405	5.10	13827	0.96	25598
1987	3969	5.60	11739	0.94	33210
1988	3225	5.69	22085	0.99	40116
1989	2524	4.63	16075	0.87	25671
1990	2807	3.64	19463	0.9	27724
1991	1572	4.43	14918	0.87	19936
1992	2269	4.14	15376	0.82	22038
1993	2110	1.77	15189	0.68	14033
1994	2507	2.90	19744	0.77	22412
1995	3127	3.21	19275	0.61	21795
1996	3113	3.66	20189	0.69	25324
1997	3681	2.80	19960	0.97	29668
1998	863	3.53	15538	0.76	14855
1999	3961	4.39	22293	0.87	36784
2000	3520	4.21	29240	0.75	36749
2001	2854	4.38	25812	1.67	59614

Sources: Columns 2,4: DOE,: *U.S. Crude Oil, Natural Gas, and Natural Gas Liquids Reserves, 2001 Annual Report* , Table 1]; 2001 from DOE release. Columns 3,5: Adelman and Watkins [2002b]; column 6 = col 2 x col 3 + col 4 x col5.

Table 2
Aggregated Oil and Gas Reserve Additions Indexed to 1982

1 *Year*	2 *Divisia Index*	3 *BOE Index Thermal Conversion Factor*	4 *BOE Index Price Ratio Conversion Factor*
1982	1.00	1.00	1.00
1983	1.31	1.28	1.44
1984	1.30	1.28	1.43
1985	1.19	1.16	1.32
1986	0.94	0.91	0.95
1987	1.17	1.13	1.29
1988	1.37	1.34	1.36
1989	1.03	1.01	1.04
1990	1.20	1.18	1.19
1991	0.82	0.79	0.77
1992	0.97	0.94	0.96
1993	0.94	0.90	0.91
1994	1.18	1.13	1.12
1995	1.27	1.23	1.27
1996	1.30	1.26	1.29
1997	1.38	1.35	1.42
1998	0.77	0.68	0.61
1999	1.64	1.48	1.55
2000	1.82	1.64	1.62
2001	1.56	1.40	1.36

Comparison of Changes in Quantities. Table 2 shows the quantities of aggregate reserve additions indexed to base 1982 for the three techniques: Divisia, BOE thermal conversion (5.5), and BOE fixed price conversion (10.0).

In all years the Divisia index exceeds the thermal index. But the differences are quite modest until the latter half of the 1990s; after 1997 they become appreciable. In 12 years out of 19 the Divisia index exceeds the price ratio index; the contrary years are almost all early on. Beyond 1997 the difference between the Divisia numbers and the fixed coefficient approach is marked. In the last year (2001) the Divisia quantity index is 1.56 (relative to 1982); under thermal conversion the quantity index is 1.40; and price ratio conversion index is 1.36, which happens to be quite close to the thermal index.

Annual percentage *changes* in these indexed quantities are shown in Table 3 and plotted in Figure 1. We observe that the changes are closely correlated in some years, not in others. Over the 19 years, there is no difference in sign between the Divisia and the two fixed coefficient series, but 11 years show marked differences in percentage changes. The year over year changes oscillate considerably, as is quite common for reserve additions. The Divisia changes are more closely correlated with the thermal index changes than with those for the price ratio index.

Table 3
Annual Changes in Aggregated Reserve Quantity Indices
(%)

1 *Year*	2 *Divisia Index*	3 *BOE Index Thermal Conversion Factor*	4 *BOE Index Price Ratio Conversion Factor*
1982	---	---	---
1983	30.61	27.67	43.73
1984	-0.23	-0.11	-0.36
1985	-8.76	-9.41	-7.74
1986	-21.15	-21.14	-28.06
1987	24.34	24.08	35.78
1988	17.26	18.63	5.65
1989	-24.76	-24.77	-23.96
1990	16.91	16.51	15.05
1991	-31.52	-32.48	-35.54
1992	17.45	18.21	24.24
1993	-3.26	-3.81	-4.67
1994	26.58	25.15	23.49
1995	7.48	8.77	12.79
1996	2.35	2.29	1.53
1997	6.18	7.76	10.62
1998	-44.12	-49.55	-57.43
1999	112.78	117.30	156.14
2000	10.60	10.26	4.10
2001	-14.05	-14.59	-15.65

Comparison of Changes in Cost. Annual percentage changes in unit costs of aggregate reserve additions are calculated by dividing the annual percentage changes in total expenditures (col (6), Table 1) by the annual percentage changes in aggregated quantities (Table 3). Table 4 shows the annual percentage changes in estimates of unit costs of aggregated oil and gas reserve additions under the three aggregation techniques. Figure 2 provides a plot.

Differences among changes in unit costs mirror those for the quantity indexes shown in Table 3, since changes in aggregate expenditures are invariant to aggregation of the quantities. Hence variations in unit costs calculated using the Divisia index are reasonably correlated with those under the thermal conversion index, but poorly correlated with those derived from the price ratio index.

In year 2001, relative to 1982, the unit cost of aggregate oil and gas reserve additions under the Divisia approach is estimated as 1.23, that for the thermal conversion series is 1.37, and for the price ratio conversion series is 1.41. In this year, then, Divisia estimated costs of reserve additions are some 11 per cent below those estimated using the thermal aggregation, and some 15 per cent below those employing price ratio aggregation.

Figure 1
Annual Changes in Aggregated Reserve Quantity Indices

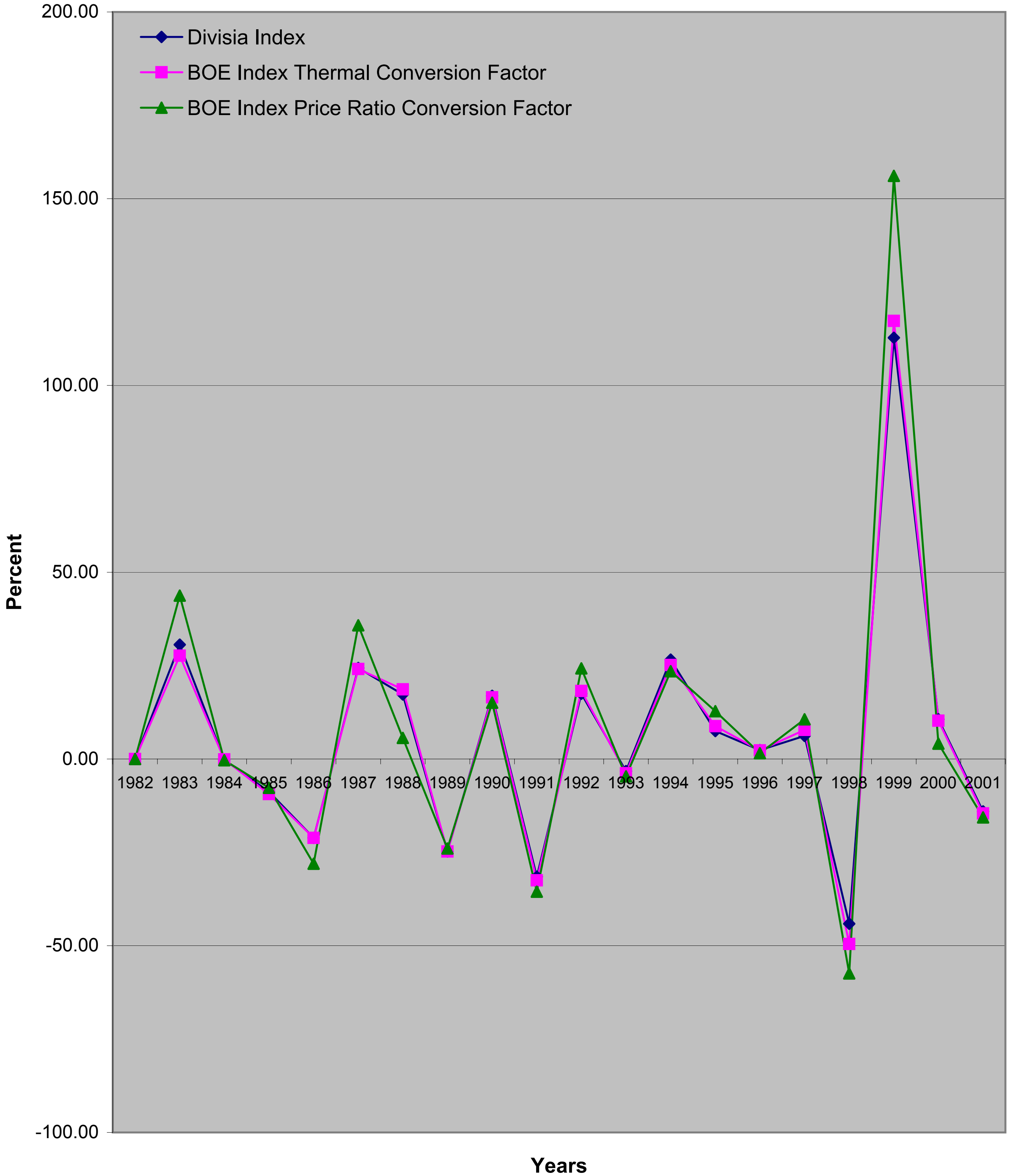


Table 4
Annual Changes in Unit Costs of Aggregated Reserve Additions
(%)

1 *Year*	2 *Divisia Quantities*	3 *BOE Quantities Thermal Conversion Factor*	4 *BOE Quantities Price Ratio Factor*
1982	---	---	---
1983	-26.24	-24.55	-32.97
1984	50.95	50.77	51.14
1985	-21.92	-21.36	-22.78
1986	8.25	8.25	18.66
1987	4.34	4.56	-4.45
1988	3.02	1.83	14.34
1989	-14.94	-14.93	-15.84
1990	-7.63	-7.31	-6.13
1991	5.01	6.51	11.57
1992	-5.88	-6.49	-11.03
1993	-34.18	-33.80	-33.21
1994	26.17	27.62	29.33
1995	-9.52	-10.59	-13.78
1996	13.53	13.58	14.44
1997	10.34	8.72	5.90
1998	-10.40	-0.75	17.62
1999	16.37	13.95	-3.33
2000	-9.67	-9.39	-4.03
2001	76.05	77.16	79.40

Comparisons of Unit Costs. Under the two fixed coefficient approaches unit costs of aggregated reserve additions can be calculated in a straightforward way: divide total expenditures from column (6) of Table 1 by the respective aggregate reserve quantities of barrels of oil equivalent. Expression of costs in this way provides a seemingly recognizable measurement of dollars per barrel, even if the quantities on which they are predicated are economic fictions.

To provide a *numerical* comparison of unit costs derived from the fixed coefficient approach we create analogous Divisia quantities, rather than changes in quantities, by applying the Divisia quantity index first to the 1982 aggregate quantity of 5399 boe derived using the thermal conversion coefficient, and second to the 1982 aggregate quantity of 3985 derived using the price ratio conversion coefficient [19]. Measurement from these common points of departure ignores any inherent differences in the base year.

[19] 5399 = 2256 + 17288/5.5; 3985 = 2256 + 17288/10 (see Table 1).

Figure 2

Annual Changes in Unit Costs of Aggregated Reserve Additions

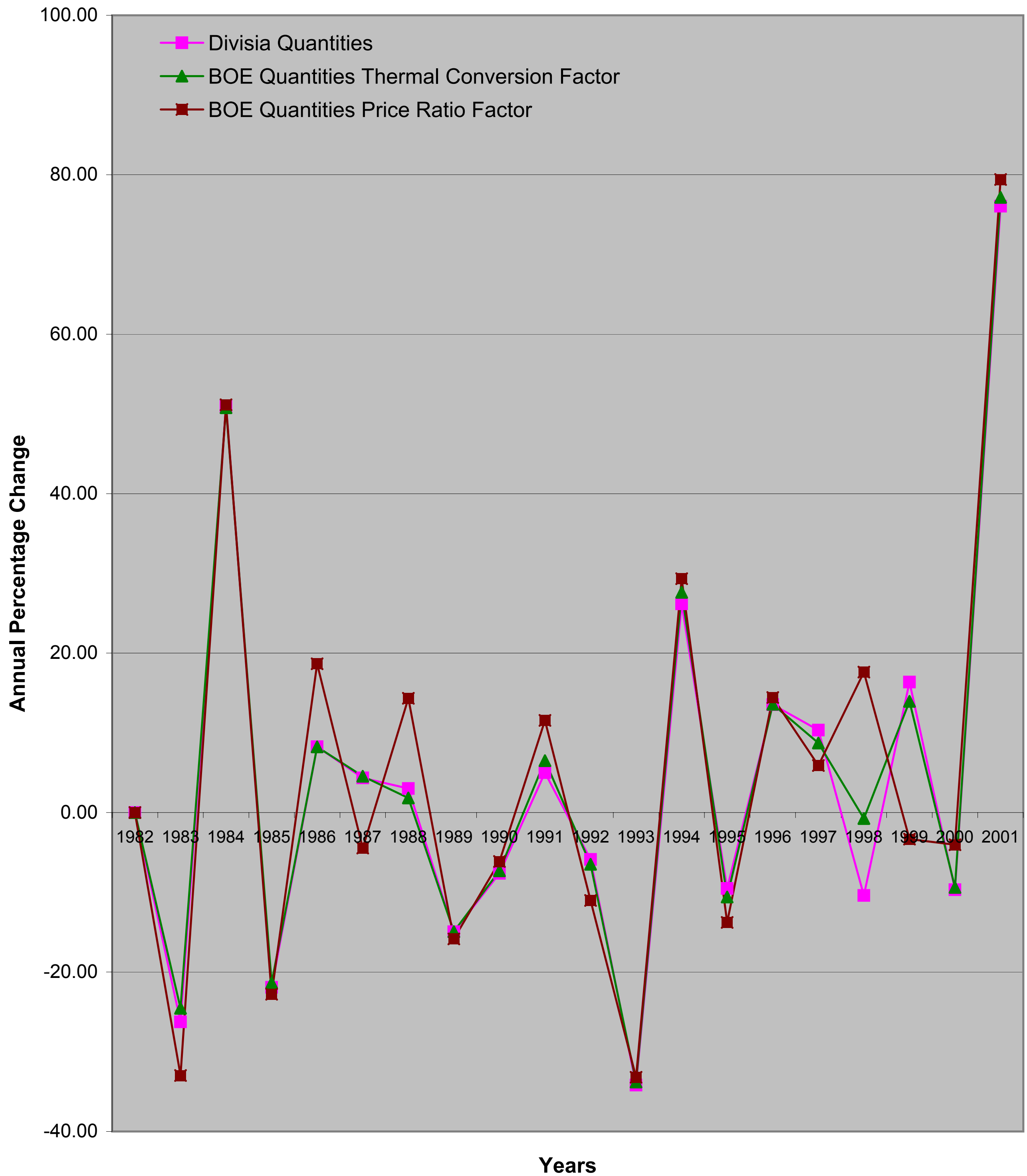


Table 5
Ratio of BOE to Divisia Estimates of Unit Cost of Aggregated Reserve Additions, Referenced to 1982

1 *Year*	2 *Ratio of BOE Thermal Conversion to Divisia Unit Cost*	3 *Ratio of BOE Price Ratio Conversion to Unit Cost*
1982	1.00	1.00
1983	1.02	0.91
1984	1.02	0.91
1985	1.03	0.90
1986	1.03	0.99
1987	1.03	0.90
1988	1.02	1.00
1989	1.02	0.99
1990	1.02	1.01
1991	1.04	1.07
1992	1.03	1.01
1993	1.04	1.03
1994	1.05	1.05
1995	1.04	1.00
1996	1.04	1.01
1997	1.02	0.97
1998	1.13	1.27
1999	1.11	1.06
2000	1.11	1.12
2001	1.12	1.15

Figure 3 plots the ratio of unit costs derived under thermal conversion to corresponding Divisia unit costs, and the parallel calculation for the price ratio conversion case. The numbers are listed in Table 5.

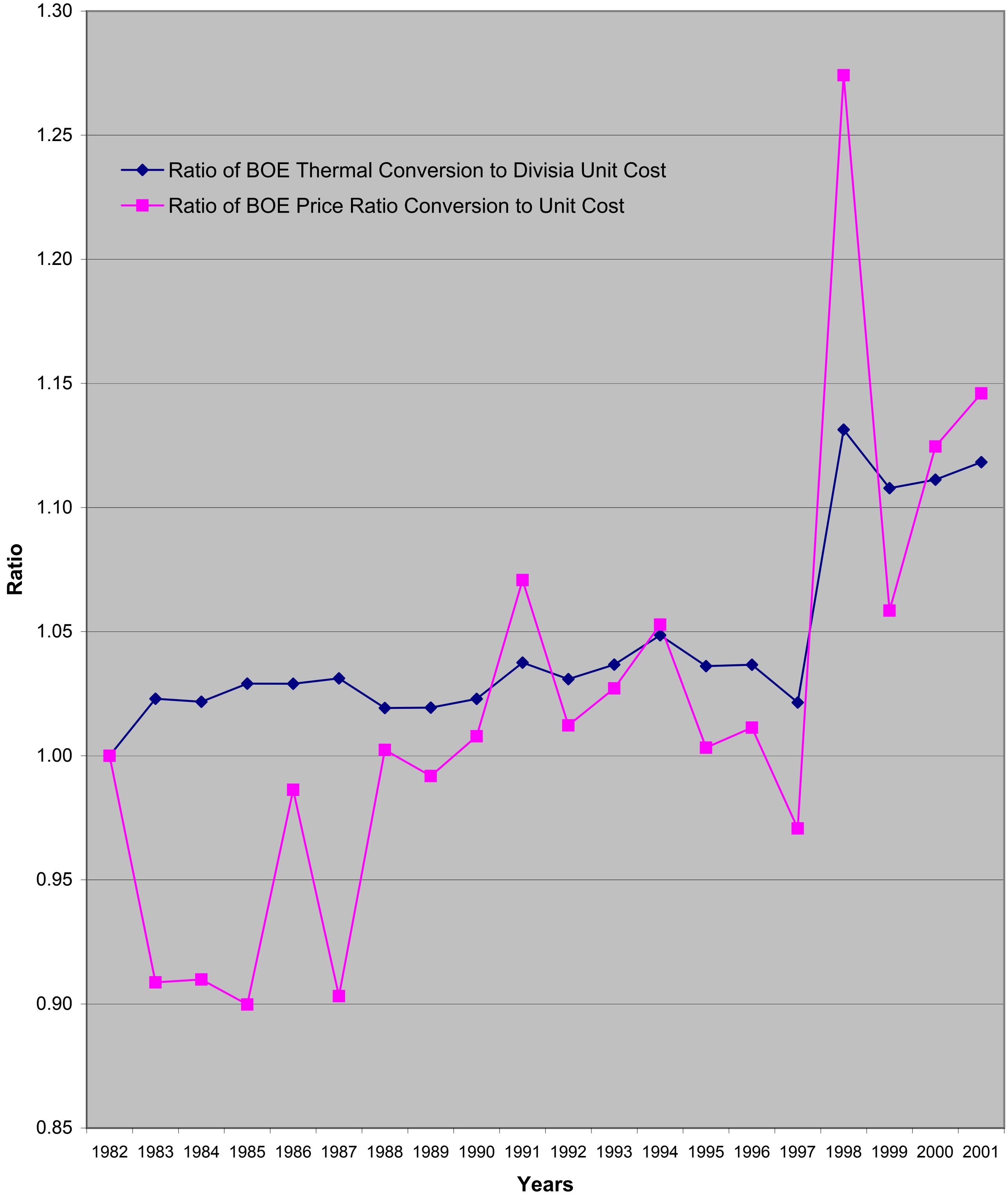
The first plot in Figure 3 shows unit costs derived using Divisia quantities (dboe) as persistently below those using the thermal conversion approach (boe), albeit only modestly before 1998. More specifically, the degree to which unit costs using thermal conversion exceed those derived from Divisia quantities range from two to five per cent during the period 1982 to 1997 before exceeding 10 per cent after 1997. We emphasize that the lower estimates of unit costs under the Divisia aggregation technique are not a general characteristic. Rather, it reflects the particular observations that make up the data used here[20].

The second plot in Figure 3 shows the Divisia approach yielding modestly higher unit costs than the fixed price ratio approach in the 1980s. But the contrary relationship held in the 1990s, and again after 1997, Divisia unit costs were appreciably lower than under the price ratio conversion.

[20] Earlier work with different reserve prices generated Divisia unit costs often above those for the fixed coefficient approaches.

Figure 3

Ratio of BOE to Divisia Estimates of Unit Cost of Aggregated Reserve Additions, Referenced to 1982



We add that if the implicit Divisia quantity were higher in the first year, 1982, the Divisia estimate of unit costs decline further in comparison with the fixed coefficient figures (and vice-versa). For purposes of this illustration we assumed equivalence in 1982.

Moving Averages. Another issue is the extent to which the results might be affected by the lack of precision in the relationship between reserve additions for a given year and the prices attributed to them, quite apart from the variability surrounding the prices themselves. To test for this we undertook the calculations using a two-year moving average of reserve additions and of in situ prices. As one would expect, the results do dampen oscillations. They also introduce a greater disparity between changes in unit costs under the Divisia technique and those under price ratio conversion. Differences between the Divisia numbers and the thermal conversion case tend to compress.

Conclusion. Estimates of trends in unit costs of aggregate reserve additions can be appreciably affected by the aggregation technique employed. In the US in the 1980s and the first half of the 1990s the conventional fixed conversion factors have generally produced estimates of changes in unit costs not significantly different from those generated using the Divisia technique. This result was contrary to our initial impressions. More appreciable differences, however, do emerge over the balance of our estimation period. We conclude that any fixed ratio of oil in situ to gas in situ values will soon be falsified so that only a flexible scheme like Divisia should be used. In this case, a recent trend towards higher values of gas reserves versus oil reserves is not accommodated by any fixed coefficient methodology.

By avoiding distortion of aggregate cost our methodology guards against exaggerating or underestimating costs of reserve additions and drawing false conclusions about industry profitability. Recent overestimation of aggregate costs under traditional methods implies corresponding underestimation of corporate profitability and a larger upward shift of aggregate reserve supply curves than may actually have happened. We caution, however, that these conclusions are sensitive to estimates of in situ prices employed.

4. Concluding Remarks.

Costs of finding new reserves cannot be neatly partitioned between oil and gas, since such costs are typically joint. This feature has led to attempts to define costs of oil and gas reserve additions by dividing expenditures by some kind of aggregate quantity.

The aggregation technique preferred by industry and governments has been to translate gas to oil 'equivalent' by using a fixed physical thermal conversion factor, or a factor intended to express some fixed btu value equivalence. These manipulations create economic fictions.

The validity of a thermal conversion factor rests on oil and gas being close substitutes over all end uses. Differences in BTU prices of oil and natural gas, upstream and downstream, demonstrate the assumption is false. A fixed value related ratio is also false, given fluctuations in relative prices over time.

A preferable aggregation technique is one that embraces changes in economic information, such as the Divisia approach. It has been widely used in the analysis of energy demand. It can also be applied to aggregating energy supply.

Comparisons of annual changes in apparent unit costs of aggregated oil and gas reserve additions in the United States, 1982-2001, show that different techniques of aggregation can lead to significant variations in results. Over this time interval, Divisia aggregation suggests lower costs of reserve additions over recent years than those from either of the other two methods. However, this result is not a general property of the Divisia method – it follows the data.

As long as relative prices of oil and gas reserves show significant variation, the Divisia technique is preferable to a fixed coefficient approach, whether based on thermal properties or on something else.

References

Adelman, M.A and G.C.Watkins “Reserve Asset Values and the Hotelling Valuation Principle: Further Evidence.” *Southern Economic Journal* 61, No. 3 (January 1995).

Adelman, M.A. and G.C.Watkins “The Value of United States Oil and Gas Reserves.” MIT Center for Energy and Environmental Policy, MIT-CEEPR 96-004 WP, May 1996.

Adelman, M.A. and G.C.Watkins “Oil Equivalence: A Recurring Fable” IAEE 2002 International *Conference Proceedings*, May 2002.[2002a]

Adelman, M.A. and G.C.Watkins “The Value of United States Oil and Gas Reserves: An Update.” MIT Center for Energy and Environmental Policy, forthcoming, 2002 [2002b].

Berndt, E.R. “Aggregate Energy, Efficiency and Productivity Measurement”, *Annual Review of Energy*, 1978, 3: 225-73.

Berry, K.T. , T. Hasan and D. O’Bryan “Relative Information Content of Proven Reserves: The BOEs-Revenue Versus BOEs-Energy”, *Journal of Energy Finance & Development*, Vol 3, Number 1, 1998, pp 1-11.

Diewart, W.E. “Exact and Superlative Index Numbers”, *Journal of Econometrics*, 1976 4: 115-45

Fisher, I. *The Making of Index Numbers*, Boston, Houghton-Mifflin, 1922.