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Abstract

We consider an infinitely-repeated oligopoly in which at each period firms not only serve the spot market by either competing in prices or quantities but also have the opportunity to trade forward contracts. Contrary to the pro-competitive results of finite-horizon models, we find that the possibility of forward trading allows firms to sustain collusive profits that otherwise would not be possible. The result holds both for price and quantity competition and follows because (collusive) contracting of future sales is more effective in deterring deviations from the collusive plan than in inducing the previously identified pro-competitive effects.

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1 Introduction

It is generally believed that forward trading makes commodity markets more competitive by inducing firms to behave more aggressively in the spot market (e.g., Allaz, 1992; Allaz and Vila, 1993). The mere possibility of forward contracting of production forces firms to compete both in the spot and forward markets, creating a prisoner's dilemma for firms in that they

voluntarily sell forward contracts (i.e., take short positions in the forward market) and end up worse off than in the absence of the forward market. Based on this argument, forward trading has been advanced, for example, as an important mechanism to mitigate eventual market power problems in electricity markets (e.g., Joskow, 2003; Rudnick and Montero, 2002).[1]

In a recent paper, Mahenc and Salanie (2004) challenge this pro-competitive view arguing that the history of alleged manipulation on forwards markets is not consistent with Allaz and Vila's (1993) prediction of firms taking short positions in the forward market. Assuming that firms compete in prices instead of quantities in the spot market, Mahenc and Salanie (2004) show that in (subgame perfect) equilibrium firms buy part of their own productions forward (i.e., take long positions), which indeed leads to higher prices than had the forward market not existed.

Despite the radically different predictions regarding the role of forward trading in enhancing competition, the models of Allaz and Vila (1993) and Mahenc and Salanie (2004) are two sides of the same "two-period competition" coin: today, firms simultaneously choose their investments in the forward market and tomorrow, and after observing their forward investments, they compete in either quantities or prices in the spot market. Recognizing that selling forward contracts is a tough investment in the sense that lowers rival's profit all else equal, the strategic-investment models of Bulow et al (1985) and Fudenberg and Tirole (1984) would predict that is subgame perfect for firms to over-invest in forwards (i.e., go short) when they compete in quantities and to under-invest in forwards (i.e., sell fewer forwards or go long) when firms compete in prices. Therefore, if we believe that firms compete in quantities in the spot market, as in many commodity markets such as electricity markets, the pro-competitive effect of forward trading found by Allaz and Vila (1993) remains valid.

This pro-competitive effect rests, however, on the assumption that firms interact for a finite number of times (at least two times: first in the forward market and then in the spot market). In this paper we view firms as repeatedly interacting in both the forward and spot markets. At each forward market opening firms have the opportunity to trade forward contracts for delivery in any future spot market and at each spot market opening we allow them to compete in either prices or quantities. As we shall see, our dynamic competition model is entirely consistent with the evidence pointed out by Mahenc and Salanie (2004) in the sense that we can have

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[1]It has also been argued that forward trading can make a market more contestable because both incumbents and potential entrants can compete in the forward market while only incumbents compete in the spot market (see Newbery, 1998). In this paper we do not consider entry threats.

firms supplying quantities (e.g., coffee) to the spot market and, at the same time, taking long positions in the forward market in order to sustain collusive prices.[2]

It is well known that players cannot sustain cooperation in the single-period prisoners' dilemma game but they can do so in the infinitely-repeated game if they are sufficiently patient (see, e.g., Tirole, 1988). For that reason we do not question in this paper the fact that the equilibrium outcome from a finite-interaction in the market is more competitive (or at least equally competitive) than that from a repeated interaction. We are interested in a fundamentally different question that is whether the introduction of forward trading makes also firms' repeated interaction more competitive. Since the pro-competitive effect of forward contracting is still present in a repeated interaction, the possibility that forward trading could make it more difficult for firms to sustain collusion remains a possibility.

The main result of the paper, however, is that the introduction of forward trading allows firms to sustain (non-cooperative) collusive profits that otherwise would not be possible. More specifically, forward trading expands the range of discount factors for which maximal collusion can be sustained in equilibrium. The result holds under both price and quantity competition in the spot market and is the net effect of two opposing forces. On the one hand, forward contracting of future sales makes it indeed more difficult for firms to sustain collusion because it reduces the remaining non-contracted sales along the collusive plan. This is the pro-competitive effect of forward trading. On the other hand, it becomes less attractive for firms to deviate from the collusive plan for two reasons: contracting sales reduces the market share that a deviating firm can capture in the deviation period and allows for a punishment that is never milder than that in the pure-spot game. This is the pro-collusion effect. Take, for example, price competition in the spot market. Forward trading does not alter the punishment path (competitive pricing) but it does lower the profits in the deviation period because the deviating firm captures only the fraction of the spot market that was not contracted forward.

The amount of collusive forward contracting is endogenously determined, and its level can always be such that the pro-collusion effect dominates the pro-competitive effect. In fact, firms may sell no forwards in equilibrium but the threat of falling into a situation of substantial contracting is what deter firms from cheating on their collusive plan. It is interesting to observe

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[2]Our model also accommodates the tacit collusion story of Sweeting (2004) for the UK power pool in that we can simultaneously have firms competing in quantities in the spot market while taking short positions (for the near term) in the forward market. It will be shown that a moderate amount of forward sales can still attend some demand for hedging without affecting collusion possibilities relative to those in the absence of forward trading.

that in their effort to sustain collusion, firms' positions in the forward market do not follow the static-competition logic of Bulow et al. (1985) and Fudenberg and Tirole (1984), but quite the opposite. In fact, when firms compete in prices in the spot market, the critical discount factor above which firms can sustain maximal collusion is decreasing in firms' short positions (up to a certain level). Conversely, when firms compete in quantities, the critical discount factor is increasing in firms' short positions, and consequently, firms may well end up buying forwards in order to sustain collusion.

It is true, however, that if firms are exogenously required (by some regulatory authority, for example) to maintain a substantial amount of forward sales (i.e., short positions), the pro-competitive effect can dominate the pro-collusion effect making it harder for firms to sustain collusion relative to the no-forward case. The rest of the paper is organized as follows. In the next section we reproduce the pro-competitive (static) result of Allaz and Vila (1993), which is essential in constructing the punishment path for the quantity competition case. In Section 3, we study two infinitely repeated interactions. We consider first the case in which firms serve the spot market by setting prices and then the case in which they choose quantities. We conclude in Section 4.[3]

2 The finite-horizon pro-competitive result

To understand the implications of forward trading in an infinitely repeated interaction it is useful to start by considering a finite-horizon game of only two periods. This case also introduces the notation that we will use in the rest of the paper. The equilibrium solutions presented in this section were first documented by Allaz and Vila (1993).

We consider two symmetric firms (1 and 2)[4] producing a homogeneous good at the same marginal cost, c. In the first period, the two firms simultaneously choose the amount of forward contracts they want to sell or buy in the forward market that call for delivery of the good in the second period. Forward transactions are denoted by f_1 and f_2 for firms 1 and 2, respectively. We adopt the convention that $f_i > 0$ corresponds to firm i selling forwards (i.e., taking a short

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[3] After the writing of the first draft of this paper, we learned about related work done by Le Coq (2004). Her model, however, is quite different from ours in that: (i) the forward market opens only once at time 0, (ii) firms are restricted to sign equal forward obligations for each and all of the spot markets, and (iii) firms are restricted to price competition in the spot market. We do not impose these assumptions in our model, which we believe do not fit well commodity markets.

[4] We see no reason for our results to (qualitatively) change if we consider more than two firms.

position) and that $f_i < 0$ corresponds to firm i buying forwards (i.e., taking a long position).[5]

The demand for forwards comes from competitive speculators and (second-period) consumers and the forward price is denoted by p_f.[6] The forward positions taken are observable and the delivery of contracts are enforceable (note that when firms take long positions, speculators take short positions). In the second period, firms attend the spot market by simultaneous choosing quantities for production q_1 and q_2 that cover their spot-market sales and the forward obligations.[7] The spot price is given by the inverse demand function $p_s = a - (q_1 + q_2)$. Since firms' payoffs in the spot market are affected by positions taken in the forward market, which in turn affects the forward price paid by speculators, the equilibrium of the game must be obtained by backward induction.

Given forward positions f_1 and f_2, firm i's payoff in the spot market is

$$\pi_i^s = p_s(q_i + q_j)(q_i - f_i) - cq_i. \tag{1}$$

Indeed, given that firm i has already contracted f_i, it is only selling $q_i - f_i$ in the spot market. If f_i is greater than q_i then the firm must buy the good from its competitor to serve its obligation or, alternatively, it can buy back its forward position at the spot price.

Using $p_s = a - (q_1 + q_2)$, the spot market Nash equilibrium is given by

$$q_i = \frac{a - c + 2f_i - f_j}{3} \tag{2}$$

$$p_s = \frac{a + 2c - f_i - f_j}{3} \tag{3}$$

As first pointed out by Allaz and Vila (1993), the spot market becomes more competitive when firms have already contracted part of their production. The reason is that the marginal revenue, $p_s'(q_i + q_j)(q_i - f_i) + p_s(q_i + q_j)$, increases with the amount of contracting, and hence, firms find it profitable to expand their production. Conversely, when firms have taken long positions in the forward market (i.e., $f_i < 0$ for $i = 1, 2$), the spot market becomes less competitive.

Obviously, in equilibrium firms do not sell any arbitrary amount of forwards. Firms and

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[5] To prevent firms from cornering the market through long positions, we assume, as Manhec and Salanie (2004), that there exist default prices that short traders can call upon to close their positions anytime spot prices rise above the default level; furthermore, these default prices are high enough that are never utilized on or off the equilibrim path.

[6] A more formal description of the game is in the appendix of Allaz and Vila (1993).

[7] Note that in a finite-horizon context forward contracting has no effect if firms set prices instead of quantities in the spot market because the equilibrium outcome is competitive pricing regardless the amount of contracting.

speculators are assumed to have rational expectations in that they correctly anticipate the effect of forward contracting on the spot market equilibrium. Thus, in deciding how many contracts to put into the forward market, firm i evaluates the following payoff function

$$\pi_i = p_f f_i + \delta \pi_i^s(f_i, f_j)$$

where $\delta < 1$ is the discount factor and $\pi_i^s(f_i, f_j)$ are the spot equilibrium profits. Rearranging terms, firm i's overall profits as a function of f_i and f_j can be written as

$$\pi_i = \delta[p_s(f_i, f_j)q_i(f_i, f_j) - cq_i(f_i, f_j)] + [p_f - \delta p_s(f_i, f_j)]f_i \tag{4}$$

where $q_i(f_i, f_j)$ and $p_s(f_i, f_j)$ are given by (2) and (3), respectively.

The first bracketed term of (4) is the standard Cournot profit while the second is the arbitrage profits. Since the presence of competitive speculators eliminate all arbitrage possibilities, i.e., $p_f = \delta p_s$, the second term is zero and the forward market equilibrium outcome is given by

$$f_i = \frac{a-c}{5} \quad \text{for} \quad i = 1, 2$$

$$q_i = \frac{2(a-c)}{5} \quad \text{for} \quad i = 1, 2$$

$$p_s = \frac{p_f}{\delta} = \frac{a+4c}{5}$$

It is clear that this outcome is more competitive than that of the standard Cournot game where firms only attend the spot market.

The mere opportunity of trading forward contracts creates a prisoner's dilemma for the two firms. Forward trading makes both firms worse off relative to the case where they stay away from the forward market. However, if firm i does not trade forward, then firm j has all the incentives to make forward sales because it would obtain a higher profit, that is, a Stackelberg profit.

This is the pro-competitive effect of forward trading, which becomes more intense as we increase the number of periods in which firms can trade forward contracts before production. Since in our infinite-horizon analysis of forward trading will also make use of the equilibrium solution for the case in which firms face more than one forward market opening, below we will present the results and refer the reader to Allaz and Villa (1993) for the proof.

Suppose then that before spot sales and production decisions are taken, there are N periods where the two firms can trade forward contracts that call for delivery of the good at the time the spot market opens. Denote these trading periods by $N, ..., k, ..., 1$ and the production period by zero (period k occurs k periods before production). As before, firms simultaneously choose f_1^k and f_2^k at period k knowing past forward sales and anticipating future forward and spot sales. In the last period, both firms simultaneously choose production levels q_1 and q_2 and the spot market clears according to the inverse demand function $p_s(q_1 + q_2)$. The per-period discount factor is δ.

The Allaz and Vila (AV) equilibrium outcome is characterized by

$$F_i^{AV}(N) = \frac{a-c}{2}\left(1 - \frac{3}{3+2N}\right) \quad \text{for } i = 1, 2 \tag{5}$$

$$q_i^{AV}(N) = \frac{a-c}{2}\left(1 - \frac{1}{3+2N}\right) \quad \text{for } i = 1, 2 \tag{6}$$

$$p_s^{AV}(N) = \frac{p_f^k}{\delta^k} = c + \frac{a-c}{3+2N} \tag{7}$$

where F_i^{AV} is firm i's aggregate forward position and p_f^k is the forward price in period k. As N tends to infinity, the non-contracted production $q_i^{AV} - F_i^{AV}$ tends to zero, the spot price tends to marginal cost and, hence, firms profits tend to zero (note that the discount factor does not affect the equilibrium solution; it only scales forward prices).

3 Repeated interaction

Consider now the infinite-horizon setting in which the same two firms repeatedly interact in both the forward and spot markets. To facilitate the exposition, we let the forward market open in the even periods ($t = 0, 2, ...$) and the spot market in the odd periods ($t = 1, 3, ...$).[8] And to facilitate the comparison with pure-spot repeated games, we let the per-period discount factor be $\sqrt{\delta}$, so the discount factor between two consecutive spot market openings is δ (or alternatively, suppose that the forward market in t opens right after the spot market in $t-1$, so the only discounting is between the forward market in t and the spot market in $t+1$, which would be δ).

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[8]Note that by including period 0 we ensure that all spot markets are preceded by a forward market where firms have the opportunity to sell forward contracts.

We will denote by $f_i^{t,t+k}$ the amount of forward contracts sold by firm i at time t that calls for delivery in the spot market that opens k periods later, i.e., at time $t + k$, where $k = 1, 3, 5...$ Notation on demand and costs are as previously defined. In addition, we denote the price, quantity and profit associated to the one-period monopoly solution by $p^m = (a+c)/2$, $q^m = (a - c)/2$ and $\pi^m = (p^m - c)q^m = (a - c)^2/4$, respectively. We will allow firms to attend the spot market by either setting quantities or prices.[9] Because it is simpler, we will study the latter case first.

3.1 Price competition in the spot market

Consider the case in which firms serve the spot market by simultaneously setting prices p_1^t and p_2^t. When firms charge different prices the lower-price firm gets the whole (spot) market, and when they charge the same price they split the market. We know for the pure-spot game that the one-period Bertrand equilibrium $p_1 = p_2 = c$ is an equilibrium of the infinitely repeated game for any value of the discount factor δ. More interestingly, we know that via trigger strategies firms can sustain the monopoly outcome $p_1^t = p_2^t = p^m$ in a subgame-perfect equilibrium as long as $\delta \geq 1/2$ (Tirole, 1988).[10]

Let us now explore the effect that forward trading has on the critical value of the discount factor for which firms can sustain the monopoly outcome in a subgame-perfect equilibrium. In doing so, we will first focus on the case in which firms take short positions (i.e., sell forwards) in the forward market and then on the case in which they take long positions (and speculators take short positions).

Consider then, the following (symmetric) trigger strategies in which firms are partially or fully contracted only one period ahead:[11] In period 0, firm i sells $f_i^{0,1} = xq^m/2$ and $f_i^{0,k} = 0$ for all $k > 1$, where $0 \leq x \leq 1$. Depending on whether t corresponds to a forward or spot opening, firm i operates as follows: If t is an odd period (i.e., spot opening), firm i sets $p_i^t = p^m$ if in every

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[9]We will also discuss the possibility of price setting in the forward market. Note that the latter can be interpreted as firms simultaneously choosing quantities of forward contracts under Bertrand conjectures (see, e.g., Green, 1999).

[10]The pair of (symmetric) trigger strategies are defined as follows: Firm i charges p^m in period 0. It charges p^m in period t if in every period preceding t both firms have charged p^m; otherwise it sets its price at marginal cost c forever after.

[11]This short-term contracting is common in the UK electricity pool. In fact, Green (1999) explains that most large customers sign one-year contracts rather than multi-year contracts in the annual contract round during the winter. Future production contracting in the copper industry exhibits a similar pattern of one-year contracting. Unlike the UK pool, in this case only a fraction of the price is contracted in advance, the rest is indexed to the spot price prevaling at the time of delivery.

period preceding t both firms have charged p^m (in the odd periods) and have forward contracted $xq^m/2$ one period ahead (in the even periods); otherwise firm i sets its price at marginal cost c forever after. If t is an even period (i.e., forward opening), firm i sells $f_i^{t,t+1} = xq^m/2$ and $f_i^{t,t+k} = 0$ for all $k > 1$ if in t and every period preceding t firms have charged p^m and have forward contracted $xq^m/2$ one period ahead; otherwise firm i sells any arbitrary amount of forward contracts (not too large so prices do not fall below marginal costs; more precisely, $f_i + f_j \leq a - c$).

Proposition 1 *The above strategies constitute a subgame perfect equilibrium if $\delta \geq \underline{\delta}(x)$, where*

$$\underline{\delta}(x) = 1 - \frac{2}{(2-x)^2 + 2x} \leq \frac{1}{2} \quad for \ all \ \ 0 \leq x \leq 1$$

To demonstrate this proposition we will first show that $\underline{\delta}(x)$ is the critical discount factor when the equilibrium level of contracting is x and then that this critical value is no greater than $1/2$. We know that the punishment phase (i.e., reversion to static Bertrand forever) is subgame perfect,[12] so it remains to find the condition under which deviation from the collusive path is not profitable for either firm. In principle, a firm can deviate by either undercutting its spot price (not necessarily by an arbitrarily small amount, as will become clear shortly) or increasing its forward sales. The latter, however, is never profitable because any deviation in the forward market is instantly detected by speculators who will pay no more than the next period spot market price, i.e., the marginal cost c.

Thus, we need only concentrate on deviations in the spot market. Given that at the opening of the spot market in period t there is an already secured supply of xq^m units coming from firms' forward obligations signed in $t-1$, firm i's optimal deviation is not to charge $p^m - \varepsilon$ as in the pure-spot case (with ε infinitesimally small), but rather to charge

$$p_i^d = \arg\max_p \left\{ (p-c)(a - xq^m - p) \right\} = \frac{a + c - xq^m}{2}$$

and supply an extra amount of $q_i^d = (a - c - xq^m)/2$, yielding profit in the deviation period of

$$\pi_i^d = (p_i^d - c)q_i^d = \frac{(a - c - xq^m)^2}{4}$$

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[12] We know from Abreu (1988), that the threat of Nash reversion does not necesarily provide the most severe credible punishment; but in this case it does. Note also that even in the absence of storage costs there are no incentives to store production because of the declining price structure (in present value terms) along either the collusive phase or the punishment phase.

Since there are no profits along the punishment phase, which starts at the next forward opening in $t + 1$, the deviation payoff is simply π_i^d.

On the other hand, firm i's continuation payoff at the opening of the spot market in t includes the non-contracted fraction of the monopoly sales of that period, i.e., $(1-x)\pi^m/2$, and the present value of the monopoly sales for the remaining periods, i.e., $\delta\pi^m/2(1-\delta)$.[13] Hence, firm i will not have incentives to deviate as long as

$$\frac{(1-x)\pi^m}{2} + \frac{\delta\pi^m}{2(1-\delta)} \geq \pi_i^d + 0 + ... \tag{8}$$

Replacing π^m and π_i^d into (8), we obtain that collusion can be sustained in equilibrium if $\delta \geq \underline{\delta}(x)$. Furthermore, the critical discount factor $\underline{\delta}(x)$ is strictly decreasing in the level of contracting from $\underline{\delta}(x = 0) = 1/2$ to $\underline{\delta}(x = 1) = 1/3$. In the Appendix A, we show that this results does not hinge on the linear demand assumption. Note also that if firms go long in the forward market, i.e., $x < 0$, the discount factor needed to sustain collusion is above $1/2$.

Contrary to the pro-competitive results of finite-horizon games, Proposition 1 indicates that forward trading allows firm to sustain collusive profits beyond what would be feasible in the absence of forward markets. The logic behind this result is simple. By allowing firms to contract part of their sales in advance, forward trading reduces firms' continuation payoffs along the collusive path (LHS of (8)), which increases the incentives for any firm to cheat on the collusive agreement. Together with this pro-competitive effect, however, forward trading also reduces firms' payoffs from deviation (RHS of (8)) because the deviating firm no longer gets the entire market in the period of deviation.

Proposition 1 also indicates that the level of contracting required to sustain the collusive outcome may not be any arbitrary number. In fact, if the discount factor is $1/3$, the only way for firms to sustain monopoly profits is by fully contracting just one period ahead (increasing contracting beyond one-period ahead reduces the continuation payoff without altering the deviation payoff). If, on the other hand, the discount factor is $1/2$, the equilibrium level of contracting can vary from zero contracting, to full contracting for exactly two periods ahead,[14] to partial contracting for more than two periods ahead. More generally, since the level of contracting is something that can be chosen, collusive contracting levels never leave firms worse off

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[13] The production costs associated to the contracted quantities are not considered because they cancel out in the deviation condition (8), i.e., these costs are incurred by the firm regardless whether it deviates or not.

[14] With two-periods ahead of full contracting, eq. (8) becomes $\delta^2\pi^m/2(1-\delta) \geq \pi_i^d$.

than in the absence of forward markets.[15] For example, we can very well have firms signing no contracts in equilibrium, which would not occur in a finite-horizon setting.

Consider now the case in which firms take long positions (i.e., buy forwards) in the forward market. If we take the same (symmetric) trigger strategies described above but for $-1 \leq x < 0$, it is possible to establish

Proposition 2 *The strategies in Proposition 1 constitute a subgame perfect equilibrium if $\delta \geq \underline{\delta}(x) = 1/2$ for all $-1 \leq x < 0$.*

This Proposition indicates that going long in the forward market does not affect firms' ability to sustain the collusive outcome compared to the situation without forwards. Following the demonstration of Proposition 1, the demonstration of Proposition 2 reduces to checking firms' incentives to deviate in the spot market. Having each firm bought long $xq^m/2$ in $t - 1$, at the opening of the spot market in period t there is an additional "demand" of xq^m units coming from speculators who need to cover their short positions taken in $t - 1$. Hence, firm i's optimal deviation is to charge $p^m - \varepsilon$ as in the pure-spot case (with ε infinitesimally small) and take the entire spot market. Since speculators will be closing their positions at the same monopoly price regardless of whether a deviation occurs or not, condition (8) reduces to (recall that forward positions do not imply additional output being effectively supplied to the market)

$$\frac{\pi^m}{2(1-\delta)} - \frac{xp^m q^m}{2} \geq \pi^m - \frac{xp^m q^m}{2} + ...$$

which is the pure-spot deviation condition.

Before moving to quantity competition in the spot market, it is worth mentioning the implications on the equilibrium outcome of price setting instead of quantity setting in the forward market (regardless of whether firms take short or long positions). If there is price setting in the forward market or, alternatively, quantity setting with Bertrand conjectures, deviations will not occur in the forward market because the deviating firm can only sell its forward contracts at marginal cost in the period of deviation, implying that our previous results hold true under price competition in the forward market.

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[15]Obviously, the multiplicity of equilibria does not guarantee that firms do not end up worse off.

3.2 Quantity competition in the spot market

Consider now the case in which firms serve the spot market by simultaneously choosing quantities q_1^t and q_2^t. We know for the pure-spot game that the one-period Cournot equilibrium $q_1 = q_2 = (a-c)/3$ is an equilibrium of the infinitely repeated game for any value of the discount factor δ and that via trigger strategies that include reversion to Nash-Cournot in case of deviation firms can sustain the monopoly outcome in subgame perfect equilibrium as long as $\delta \geq 9/17 = 0.529$.[16]

As before, to explore the effect that forward trading has on firms' ability to sustain monopoly profits we consider the following (symmetric) strategies in which firms are partially or fully contracted only one period ahead (unlike the price setting game, here we do not need make an explicit distinction between firms taking short or long positions; nevertheless, to facilitate the exposition we think of firms as taking short positions): In period 0, firm i sells $f_i^{0,1} = xq^m/2$ and $f_i^{0,k} = 0$ for all $k > 1$, where $-1 \leq x \leq 1$. Depending on whether t corresponds to a spot or forward opening, firm i operates as follows: If t is an odd period (i.e., spot opening), firm i sets $q_i^t = (1-x)q^m/2$ if in every period preceding t both firms have chosen $(1-x)q^m/2$ (in the odd periods) and have forward contracted $xq^m/2$ one period ahead (in the even periods); otherwise firm i plays according to Allaz and Vila (AV) equilibrium thereafter. If t is an even period (i.e., forward opening), firm i sells $f_i^{t,t+1} = xq^m/2$ and $f_i^{t,t+k} = 0$ for all $k > 1$ if in t and every period preceding t firms have chosen $(1-x)q^m/2$ and have forward contracted $xq^m/2$ one period ahead; otherwise firm i follows AV thereafter.

Proposition 3 *The above strategies constitute a subgame perfect equilibrium if $\delta \geq \underline{\delta}(x)$, where $\underline{\delta}(x)$ solves*

$$\frac{[1-x+x\underline{\delta}(x)]}{8[1-\underline{\delta}(x)]} = \frac{(3-x)^2}{64} + \sum_{N=1}^{\infty} \frac{(1+N)[\underline{\delta}(x)]^N}{(3+2N)^2} \tag{9}$$

and $\underline{\delta}(x) < 9/17$ for all $-1 \leq x \leq 1$.

Proposition 3 states that under Nash-reverting punishments forward trading allows firms to sustain monopoly profits that otherwise would not be possible (i.e., when $\underline{\delta}(x) \leq \delta < 9/17$). In demonstrating this proposition, we will show that a deviation in the spot market is more attractive than a deviation in the forward market, that $\underline{\delta}(x)$ is the critical discount factor when

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[16]The no-deviation condition is $\pi^m/2(1-\delta) \geq \pi^d + \delta\pi^c/(1-\delta)$, where $\pi^d = 9(a-c)^2/64$ and $\pi^c = (a-c)^2/9$. Since reversion to the Nash-Cournot equilibrium of the stage game is not the most severe credible punishment (Abreu, 1986; 1988), we will discuss this issue at the end of the section.

the equilibrium level of contracting is $-1 \leq x \leq 1$, and that $\underline{\delta}(x)$ strictly lower than the critical discount factor of $9/17$. Since the punishment phase of reverting to AV is subgame perfect, it remains to find the condition under which deviation from the collusive path is not profitable for any firm. In principle, a firm can deviate by increasing either its forward sales (not only for the next spot market but more generally for any future spot market) or its spot sales.

Unlike the price setting game, here it is less obvious that a deviation in the forward market is less attractive than a deviation in the spot market for any level of contracting. As shown in the Appendix B, however, firm i's optimal deviation in the tth forward market for delivery in the spot market in $t+1$, when firm j is selling $f_j^{t,t+1} = xq^m/2$, is to sell $f_{id}^{t,t+1} = (a - c - xq^m/2)/4$ (the optimal deviations for delivery in each of the following spot markets are developed in the Appendix B as well). Given these forward quantities $f_j^{t,t+1}$ and $f_{id}^{t,t+1}$ and the associated spot quantities (from eq. (2)), firm i's profit in $t+1$ is $(a - c - xq^m/2)^2/8$, which is not greater than the monopoly profit of $\pi^m/2 = (a - c)^2/8$ that the firm would have received in $t+1$ had continued cooperating (recall that at the beginning of t no forward contract for delivery at $t+1$ has yet been sold). Since per-period profits along the AV punishment phase fall overtime as future spot markets are preceded by an increasing number of forward openings, it becomes clear that a firm will never find it profitable to deviate in the forward market.

We now look at firm i's incentives to deviate in the tth spot market. Given that at the opening of the spot market in t there is an already secured supply of xq^m units coming from firms' forward obligations signed in $t-1$, the firm's optimal deviation is[17]

$$q_i^d = \arg\max_q \left\{ \left(a - xq^m - \frac{(1-x)q^m}{2} - q - c \right) q \right\} = \frac{a-c}{2} - \frac{(1+x)q^m}{4}$$

so firm i's total production in the period of deviation is $q_i^d + xq^m/2 = (a-c)/2 - (1-x)q^m/4$. The clearing spot price associated to this (optimal) deviation is $p^d = (a+c)/2 - (1+x)q^m/4$, hence, profits in the period of deviation are

$$\pi_i^d = (p^d - c)q_i^d = \frac{(a-c)^2(3-x)^2}{64}$$

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[17]Note that the total production of firm i in the period of (optimal) deviation is $q_i^D = q_i^d + xq^m/2$. Some readers may find easier to solve for q_i^D, especially if firms are taking long positions, which is obtained from (see eq. (1))

$$q_i^D = \arg\max_q \left\{ \left(a - \frac{q^m}{2} - q \right) \left(q - \frac{xq^m}{2} \right) - cq \right\} = \frac{a-c}{2} - \frac{(1-x)q^m}{4}$$

which are never greater than the profits in the deviation period in the pure-spot quantity game. Note that if firms are buying their entire production for the next period (i.e., $x = -1$), no firm has incentives to increase its supply and deviate from the monopoly price $p^m = (a+c)/2$. This is because the gains from increasing market share are, at the margin, exactly offset with the losses from lowering the price at which short traders can close their positions.

After the deviation period, firms follow the punishment path given by the AV subgame-perfect equilibrium. Hence, contracting, production and price equilibrium levels corresponding to a future spot market preceded by N forward market openings, where the first opening is right after the deviation, are given by eqs. (5)–(7). Then, firm i's punishment profit associated to the spot market that is preceded by N forward openings is

$$\pi_i^p(N) = (p_s^{AV}(N) - c)q_i^{AV}(N) = \frac{(a-c)^2(1+N)}{(3+2N)^2}$$

Note that $\pi_i^p(N)$ tends to zero as N approaches infinity.

On the other hand, firm i's continuation payoff at the opening of the spot market in t includes the non-contracted fraction of the monopoly sales of that period, i.e., $(1-x)\pi^m/2$, and the present value of the monopoly sales for the remaining periods, i.e., $\delta\pi^m/2(1-\delta)$. Hence, firm i will not have incentives to deviate from the monopoly path as long as

$$\frac{(1-x)\pi^m}{2} + \frac{\delta\pi^m}{2(1-\delta)} \geq \pi_i^d + \sum_{N=1}^{\infty} \delta^N \pi_i^p(N) \tag{10}$$

Replacing π^m, π_i^d and $\pi_i^p(N)$ into (10), we obtain that maximal collusion can be sustained in equilibrium if $\delta \geq \underline{\delta}(x)$.

Contrary to the pricing game, here the critical discount factor $\underline{\delta}(x)$ is strictly increasing in the level of contracting from $\underline{\delta}(x = -1) = 0$, to $\underline{\delta}(x = 0) = 0.238$, and to $\underline{\delta}(x = 1) = 0.512 < 9/17$.[18] This is because an increase in x reduces the continuation payoff more than the one-period deviation profit (i.e., π_i^d) while it has no effect on profits along the punishment phase (it would affect them if forward contracts along the collusive path were signed for delivery beyond one period ahead and above the AV equilibrium level).

Although we have limited our analysis to collusive paths with forward contracts for only

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[18]Values for $\underline{\delta}(x)$ can only be obtained numerically since the last term of (9) is a hypergeometric serie that does not converge to a closed form. Note also that because $\underline{\delta}(x)$ is increasing in x there is no problem here if the game "starts" with a spot-market opening rather than with a forward-market opening.

one period ahead, it should be clear that in equilibrium we can observe different contracting profiles depending on the discount factor. If the discount factor is 1/2, for example, firms can sustain maximal collusion whether they are almost fully contracted for only one period ahead or partially contracted for various periods ahead. However, if $\delta = 0.238$, the only way for firms to sustain monopoly profits is by not selling any forwards. This is interesting because we can observe very little contracting in equilibrium but the threat of falling into a situation of substantial contracting is what deter firms from cheating on their collusive agreement.

Going back to the evidence pointed out by Mahenc and Salanie (2004) that in the history of alleged manipulation of commodity markets, it has been observed that unreasonable high prices resulted from long positions held by a cartel of producers, our results are consistent with such evidence. If firms are too impatient (partly driven by a low probability that firms assign to the sustainability of the cartel in the future), the only way for firms to sustain a collusive outcome is by taking long positions in the forward market.

It is important to emphasize that even if firms are not allowed to take long positions in the forward market, we have shown that firms' abilities to sustain collusion increases with the introduction of forward trading as long as the punishment strategy of the pure-spot quantity game is reversion to the Nash-Cournot equilibrium of the stage game. As demonstrated by Abreu (1986 and 1988), there exist more severe subgame-perfect punishment paths that could allow firms to sustain monopoly profits in the pure-spot quantity game for lower discount factors. These punishment paths, commonly known as (simple) penal codes, are comprised of a stick and a carrot phase. Without deriving what would be a penal code in the presence of forward trading, which seems far from a simple exercise, we can document here that the lowest discount factor for which monopoly profits can be sustained in the pure-spot quantity game through an optimal penal code is $9/32 = 0.281$.[19] The reason of why the latter is larger than $\underline{\delta}(x = 0)$ is because firms obtain lower present value profits along the AV subgame equilibrium path than along the harshest possible punishment path in the pure-spot game. This corroborates that forward trading expands the range of discount factors for which maximal collusion can be

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[19]This critical value is obtained by simultaneously solving the two no-deviation conditions (see Abreu, 1986)

$$\delta(\pi^m/2 - \pi(z)) = \pi^{dp} - \pi(z)$$

$$\delta(\pi^m/2 - \pi(z)) = \pi^d - \pi^m/2$$

where z is the "stick" quantity, π^{dp} is the one-period profit from optimally deviating in the punishment phase when the other firm is playing z, and π^d is the one-period profit from optimally deviating in the collusive phase when the other firm is playing $q^m/2$. Solving we obtain $\delta = 9/32$ and $z = 5(a - c)/12$ (the latter is the largest root from the corresponding quadratic equation).

sustained in equilibrium.

Finally, let us discuss the implications on the equilibrium outcome of price setting (i.e., quantity setting with Bertrand conjectures) instead of quantity setting in the forward market. The role of forward trading is strengthened because the punishment path is now competitive pricing (only at competitive pricing will no firm have incentives to slightly reduce the price of its forward contracts below that of its rival's). Deviations in the forward market are, as before, never profitable because the deviating firm can only sell its forward contracts at marginal cost. Deviations in the spot market, on the other hand, become less attractive, thus reducing the critical discount factor for the limiting case of no contracting in equilibrium to just $1/9$.[20]

4   Final remarks

We have studied the strategic implications of forward contracting in commodity markets that exhibit an oligopolistic structure and where firms repeatedly interact in both spot and forward markets.[21] Unlike the pro-competitive effects found in static models that restrict firms inter-action to a finite number of periods, we have found that the mere possibility of (voluntary) forward trading allows firms to sustain collusive profits that otherwise would be impossible. This is because the contracting of future sales can be made more effective in deterring deviations from the collusive plan than in inducing the previously identified pro-competitive effects. More precisely, the introduction of forward markets expands the range of discount factors for which maximal collusion can be sustained in equilibrium.

The above results apply for both price and quantity competition in the spot market but the use of forward trading to sustain collusion depends critically on the type of competition. When firms compete in prices, the critical discount factor above which firms can sustain maximal collusion is decreasing in firms' short positions (up to a certain level), so firms would need to sell forward in order to face a lower critical discount factor than that in the absence of forward markets. Conversely, when firms compete in quantities, the critical discount factor is increasing in firms' short positions, and consequently, firms would need to sell little forwards or perhaps buy forwards in order to sustain collusion.

Recognizing that selling forward contracts can be interpreted as a tough investment in the

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[20]The continuation payoff is $\pi^m/2(1-\delta)$ while the deviation payoff is $9(a-c)^2/64 + 0 + ...$.

[21]See Anderson (1984) for a discussion of forward trading activity in (imperfectly competitive) commodity markets.

sense that it lowers rival's profit all else equal, in their effort to sustain collusion, firms' positions in the forward market follow the exact opposite logic of the (static) models of Allaz and Vila (1993) and Mahenc and Salanie (2004) and of the more general strategic-investment models of Bulow et al. (1985) and Fudenberg and Tirole (1984). As formally shown in the Appendix C, the reason for this investment-reversion result is because the sign of the strategic effect, as identified by Bulow et al. (1985) and Fudenberg and Tirole (1984), works in the exact opposite direction in our dynamic competition story. Whether this result applies more generally to other cases in which firms make frequent investments (e.g., marketing) and at the same time compete in the spot market (in either prices or quantities) remains to be investigated.

The results of the paper can have important policy implications, particularly in markets where firms repeatedly interact and where forward contracting is viewed as a important mechanism to mitigate eventual market power problems. Electricity markets are good examples. Since we show in the paper that voluntary forward contracting need not lead to more competitive outcomes, one might be tempted to prescribe that the regulatory authority should require a minimum amount of contracting sufficient enough that the pro-competitive effect of forward contracting dominates its pro-collusion effect. Unless this minimum amount is large enough (which may render the measure impractical), introducing a minimum amount of contracting can have the exact opposite effect, however. It can help firms to "disregard" more competitive equilibria by serving as a focal point towards the coordination on more collusive equilibria (Knittel and Stango, 2003).[22]

Since there is virtually no literature on the effects of forward trading on repeated games, one can identify different areas for future research. Based on the discussion at the end of section 3.2, one obvious candidate is the study of more severe credible punishments along the optimal penal codes of Abreu (1986 and 1988). Another candidate is the extension of the price wars of Rotemberg and Saloner (1986) and Green and Porter (1984) to forward contracting. For the latter, we could also introduce imperfect observability of individual forward positions; something we have not done in this paper.

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[22]These authors provide empirical evidence, at least for the early 1980's, that the introduction of price celings in the credit cards market did not promote competition as intended but rather served as focal point for tacit collusion.

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Appendix A: Critical discount factor for a general demand function

We will demonstrate that $d\underline{\delta}(x)/dx|_{x=0} < 0$ for a general demand function $D(p)$. The monopoly price p^m and the optimal deviation p^d solve, respectively (subindex i has been omitted)

$$(p^m - c)D'(p^m) + D(p^m) = 0$$

$$(p^d - c)D'(p^d) + D(p^d) - xD(p^m) = 0$$

Replacing the one-period deviation profits $\pi_d(x) = (p^d - c)(D(p^d(x)) - xD(p^m))$ in eq. (8), the critical discount factor becomes

$$\underline{\delta}(x) = \frac{2\pi_d(x) - (1-x)\pi^m}{2\pi_d(x) + x\pi^m}$$

and taking derivative with respect to x leads to

$$\frac{d\underline{\delta}(x)}{dx} = \frac{[\pi^m]^2 + 2\pi^m\pi_d'(x)}{[2\pi_d(x) + x\pi^m]^2} \tag{11}$$

But from the envelop theorem, we know that $\pi_d'(x) = (p^d - c)(-D(p^m))$, which evaluated at

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$x = 0$ gives $\pi'_d(x = 0) = -\pi^m$. By replacing the latter in (11), we finally obtain $d\underline{\delta}(x)/dx|_{x=0} = -1/4$.

Appendix B: Optimal deviation in the forward market

If firm i decides to deviate at the opening of the forward market in t it will do so by increasing its contract sales for delivery not only in the spot market in $t + 1$ but also in all future spot markets. We will first derive firm i's optimal forward deviation for deliveries in $t+1$ and then for deliveries for future spot markets. Given that firm j sells $f_j^{t,t+1} = xq^m/2$ forwards for delivery at $t + 1$, firm i's optimal deviation $f_{id}^{t,t+1}$ at the opening of the tth forward market maximizes

$$\pi_i^{df} = q_i(f_{id}^{t,t+1}, f_j^{t,t+1})(p_s(f_{id}^{t,t+1}, f_j^{t,t+1}) - c)$$

where $q_i(f_{id}^{t,t+1}, f_j^{t,t+1})$ and $p_s(f_{id}^{t,t+1}, f_j^{t,t+1})$ are given by (2) and (3), respectively (note that firm i's deviation is detected by speculators at the moment forward contracts are being traded). Solving, we obtain

$$f_{id}^{t,t+1} = \frac{a - c - f_j^{t,t+1}}{4}$$

yielding a profit for the deviation period equal to $\pi_i^{df}(f_j^{t,t+1}) = (a - c - f_j^{t,t+1})^2/8$.

Consider now firm i's forward sales deviation in t for deliveries in spot markets following the spot market in $t + 1$. Let then denote by $f_{id}^N \equiv f_{id}^{t,t+1+2N}$ firm i's contract sales in t for delivery in the spot market that opens in $t + 1 + 2N$, where $N \geq 1$. Note that at the opening of the forward market in $t + 2$, the spot market in $t + 1 + 2N$ will be preceded by exactly N forward openings (including the one in $t+2$). Since firm i's deviation in t is detected by firm j in the spot market in $t+1$, at the opening of the forward market in $t+2$ firms know they are in the world of the Allaz and Vila. Furthermore, given that in $t + 2$ firms observe that firm i has already contracted f_{id}^N for delivery in the spot market that is preceded by N forward openings (which effectively reduces the spot demand now faced by firms by f_{id}^N), we can deduce from eqs. (6) and (7) that the (punishment) quantity and price levels in the spot-market equilibrium as a function of f_{id}^N will be

$$q_i^p(f_{id}^N, N) = \frac{a - f_{id}^N - c}{2}\left(1 - \frac{1}{3 + 2N}\right) + f_{id}^N \tag{12}$$

$$q_j^p(f_i^N, N) = \frac{a - f_{id}^N - c}{2}\left(1 - \frac{1}{3 + 2N}\right) \tag{13}$$

$$p_s^p(f_{id}^N, N) = c + \frac{a - f_{id}^N - c}{3 + 2N} \tag{14}$$

Hence, firm i's optimal forward deviation in t is

$$f_{id}^N = \arg\max_f \{q_i^p(f, N)(p_s^p(f, N) - c)\} = \frac{a - c}{4 + 2N}$$

Replacing the optimal deviation f_{id}^N into (12)–(14), we obtain

$$q_i^p(N) = \frac{a - c}{2}; \quad q_j^p(N) = \frac{(1 + N)(a - c)}{4 + 2N}; \quad p_s^p(N) = c + \frac{a - c}{4 + 2N}$$

As in the AV original equilibrium, as N tends to infinity q_j^p tends to $(a - c)/2$ and p_s^p tends to marginal cost. Interestingly, q_i^p is always at the Stackelberg level regardless of the number of forward market openings.

Appendix C: Investment-reversion

Let us merge the no-deviation conditions (8) and (10) into one no-deviation condition that solves for the critical discount factor $\underline{\delta}(x)$ as a function of x

$$\frac{(1 - x)\pi^m}{2} + \frac{\underline{\delta}(x)\pi^m}{2(1 - \underline{\delta}(x))} = \pi_i^d(x) + \frac{\underline{\delta}(x)\pi_i^p(x)}{(1 - \underline{\delta}(x))} \tag{15}$$

where the punishment path is general enough to accommodate for both type of competitions. We know, however, that when firms restrict their forward transaction to only one period ahead, profits along the punishment path are independent of the amount of contracting x. Totally differentiating (15) with respect to x and rearranging leads to

$$\frac{d\underline{\delta}(x)}{dx} \left[\pi_i^d - \pi_i^p + \frac{\pi^m}{2} \right] = (1 - \underline{\delta}(x)) \left[\frac{\pi^m}{2} + \frac{d\pi_i^d(x)}{dx} \right] \tag{16}$$

The term in brackets in the left-hand side is always positive, so the sign of $d\underline{\delta}(x)/dx$ will be governed by the sign of terms in brackets in the right-hand side. If $d\underline{\delta}(x)/dx > 0$, firms would tend to underinvest in forwards ($x \leq 0$) in order to keep $\underline{\delta}(x)$ as low as possible. On the other hand, if $d\underline{\delta}(x)/dx < 0$, firms would tend to overinvest in forwards ($x > 0$) in order to keep $\underline{\delta}(x)$ as low as possible. Thus, to support our results we should find that for some value of x, say $x = 0.5$, $d\underline{\delta}(x)/dx < 0$ under price competition and that $d\underline{\delta}(x)/dx > 0$ under quantity competition.

It is not difficult to sign $d\underline{\delta}(x)/dx$ using some of the expressions already developed in the text for $\pi_i^d(x)$. Under price competition (P), $^P\pi_i^d(x) = (a-c)^2(2-x)^2/8$ and, under quantity competition (Q), $^Q\pi_i^d(x) = (a-c)^2(3-x)^2/64$. In both cases we have that $d\pi_i^d(x)/dx < 0$, more specifically, $d^P\pi_i^d(x)/dx = -(a-c)^2(2-x)/8$ and $d^Q\pi_i^d(x)/dx = -(a-c)^2(3-x)/32$. Since we know that $\pi^m = (a-c)^2/4$, under price competition the sign of term in brackets in the right-hand side is indeed negative as long as $x < 1$. Under quantity competition, on the other hand, the sign of term in brackets in the right-hand side is positive as long as $x > -1$.

To make the connection between these results and the predictions of the strategic-investment models of Bulow et al. (1985) and Fudenberg and Tirole (1984), let us denote by x_i and x_j the amount of forward investments done by firms i and j, respectively, and by y_i and y_j the spot actions of firms i and j (which can be prices or quantities), the term $d\pi_i^d(x_i, x_j)/dx_i$ can be decomposed, as in Fudenberg and Tirole (1984), into two effects as follows

$$\frac{d\pi_i^d(x_i, x_j)}{dx_i} = \frac{\partial \pi_i^d(x_i, x_j)}{\partial x_i} + \frac{\partial \pi_i^d(x_i, x_j)}{\partial y_j}\frac{\partial y_j}{\partial x_i} + \frac{\partial \pi_i^d(x_i, x_j)}{\partial y_i}\frac{\partial y_i}{\partial x_i} \tag{17}$$

The third term of the right-hand side is zero by the envelope theorem (y_i is an optimal deviation), so we are left with two terms. The first term of the right-hand side of (17) is the direct effect and the second term is the strategic effect. The sign of the strategic effect can be expressed as

$$\text{sign}\left(\frac{\partial \pi_i^d(x_i, x_j)}{\partial y_j}\frac{\partial y_j}{\partial x_i}\right) = \text{sign}\left(\frac{\partial \pi_j^d(x_i, x_j)}{\partial y_i}\right)\text{sign}\left(\frac{\partial y_j}{\partial x_i}\right)$$

$$= \text{sign}\left(\frac{\partial \pi_j^d(x_i, x_j)}{\partial x_i}\frac{\partial x_i}{\partial y_i}\right)\text{sign}\left(\frac{\partial y_j}{\partial x_i}\right) = \text{sign}\left(\frac{\partial \pi_j^d(x_i, x_j)}{\partial x_i}\right)\text{sign}\left(\frac{\partial y_j}{\partial y_i}\right)$$

Forward contracting (going short) is a tough investment, so $\partial \pi_j^d(x_i, x_j)/\partial x_i < 0$. If firms are competing in quantities, $\partial y_j/\partial y_i < 0$, and if they are competing in prices, $\partial y_j/\partial y_i > 0$. Thus, the sign of the strategic effect in the static context of Allaz and Vila (1993) and Mahenc and Salanie (2004) is positive under quantity competition and negative under price competition. In our dynamic story, when the sign of the strategic effect is positive (negative), the sign of $d\underline{\delta}(x)/dx$ is more likely to be positive (negative), and firms have incentives to underinvest (overinvest) in forward contracting in order to sustain collusion.